香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2012年香港中學文憑考試
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2012

數學　必修部分　試卷一
MATHEMATICS　COMPULSORY PART　PAPER 1

本評卷參考乃香港考試及評核局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

只限教師參閱　　FOR TEACHERS' USE ONLY

Hong Kong Diploma of Secondary Education Examination
Mathematics Compulsory Part Paper 1

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. Use of notation different from those in the marking scheme should not be penalized.

5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

6. Marks may be deducted for wrong units (u) or poor presentation (pp).
   a. The symbol (u-1) should be used to denote 1 mark deducted for u. At most deduct ***1 mark*** for u in each of Section A(1) and Section A(2). Do not deduct any marks for u in Section B.
   b. The symbol (pp-1) should be used to denote 1 mark deducted for pp. At most deduct ***1 mark*** for pp in each of Section A(1) and Section A(2). Do not deduct any marks for pp in Section B.
   c. At most deduct 1 mark in each of Section A(1) and Section A(2).
   d. In any case, do not deduct any marks in those steps where candidates could not score any marks.

7. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Solution	Marks	Remarks
1. $\frac{m^{-12}n^8}{n^3}$		
$=\frac{n^8}{m^{12}n^3}$	1M	for $x^{-p}=\frac{1}{x^p}$ or $\frac{1}{x^{-p}}=x^p$
$=\frac{n^{8-3}}{m^{12}}$	1M	for $\frac{y^q}{y^r}=y^{q-r}$ or $\frac{y^q}{y^r}=\frac{1}{y^{r-q}}$
$=\frac{n^5}{m^{12}}$	1A	
	----------(3)	
2. $\frac{3a+b}{8}=b-1$		
$3a+b=8(b-1)$		
$3a+b=8b-8$	1M	
$3a=7b-8$	1M	for putting a on one side
$a=\frac{7b-8}{3}$	1A	or equivalent
$\frac{3a+b}{8}=b-1$		
$\frac{3a}{8}+\frac{b}{8}=b-1$		
$\frac{3a}{8}=b-\frac{b}{8}-1$	1M	for putting a on one side
$\frac{3a}{8}=\frac{7b}{8}-1$		
$3a=8\left(\frac{7b}{8}-1\right)$		
$3a=7b-8$	1M	
$a=\frac{7b-8}{3}$	1A	or equivalent
	----------(3)	
3. (a) $x^2-6xy+9y^2$		
$=(x-3y)^2$	1A	or equivalent
(b) $x^2-6xy+9y^2+7x-21y$		
$=(x-3y)^2+7x-21y$	1M	for using the result of (a)
$=(x-3y)^2+7(x-3y)$		
$=(x-3y)(x-3y+7)$	1A	or equivalent
	----------(3)	

Solution	Marks	Remarks
4. (a) The daily wage of Ada		
$= 480(1+20\%)$	1M	
$= \$576$	1A	u−1 for missing unit
(b) Let $\$x$ be the daily wage of Christine.		pp−1 for undefined symbol
$x(1-20\%) = 480$		
$x = \dfrac{480}{1-20\%}$	1M	
$x = 600$		
Thus, Christine has the highest daily wage.	1A	f.t.
Note that $\dfrac{1}{1-20\%} > 1+20\%$.	1M	
Thus, Christine has the highest daily wage.	1A	f.t.
	----------(4)	
5. Let x be the number of male guards in the exhibition centre.		pp−1 for undefined symbol
Then, the number of female guards in the exhibition centre is $(x+24)$.	1A	
$x+(x+24) = 132$	1M+1A	
$2x = 108$		
$x = 54$	1A	
Thus, the number of male guards in the exhibition centre is 54 .		
Let x and y be the numbers of male and female guards respectively.		pp−1 for undefined symbol
So, we have $x+y=132$ and $\dfrac{y}{6}-\dfrac{x}{6}=4$.	1A+1A	
Therefore, we have $x+(x+24)=132$.	1M	for getting a linear equation in x or y only
Solving, we have $x=54$.	1A	
Thus, the number of male guards in the exhibition centre is 54 .		
The number of male guards in the exhibition centre		
$= \dfrac{132-(6)(4)}{2}$	1M+1A+1A	1M for fraction + 1A for numerator + 1A for denominator
$= \dfrac{108}{2}$		
$= 54$	1A	
	----------(4)	
6. (a) $\dfrac{4x+6}{7} > 2(x-3)$		
$4x+6 > 14(x-3)$		
$10x < 48$		
$x < \dfrac{24}{5}$	1A	$x < 4.8$
$2x-10 \le 0$		
$x \le 5$	1A	
Thus, the required solution is $x < \dfrac{24}{5}$.	1M	
(b) 4	1A	
	----------(4)	

		Solution	Marks	Remarks
7.	(a)	a $= 18.1 - 6.8$ $= 11.3$	1A	
		b $= 12.1 + 3.2$ $= 15.3$	1A	
	(b)	Note that the longest time taken by the students to finish a 100 m race after the training is 15.2 s which is less than the upper quartile of the distribution of the times taken before the training.	1M	
		Thus, the claim is agreed.	1A ----------(4)	f.t.
8.	(a)	$\Delta AED \sim \Delta BEC$	1A	
		$\Delta AEB \sim \Delta DEC$	1A	
		$\frac{AE}{BE} = \frac{DE}{CE}$ $\frac{AE}{8} = \frac{15}{20}$	1M	
		$AE = 6$ cm	1A	u−1 for missing unit
	(b)	$AE^2 + BE^2$	1M	
		$= 6^2 + 8^2$ $= 10^2$ $= AB^2$		
		Thus, AC and BD are perpendicular to each other.	1A ----------(5)	f.t.
9.	(a)	Let x cm be the length of AD.		pp−1 for undefined symbol
		$\frac{(6+x)(12)}{2}(10) = 1\,020$	1M	
		$x = 11$	1A	
		Thus, the length of AD is 11 cm .		u−1 for missing unit
	(b)	CD $= \sqrt{12^2 + (11-6)^2}$	1M	
		$= 13$ cm		
		The total surface area of the prism $ABCDEFGH$ $= (12+11+13+6)(10) + \frac{(6+11)(12)}{2}(2)$	1M	
		$= 624\text{ cm}^2$	1A ----------(5)	u−1 for missing unit

只限教師參閱 FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
10. (a) The mean		
$= 18$	1A	
The median		
$= 16$	1A	
	----------(2)	
(b) (i) The mean		
$= 18$	1A	
(ii) Let a and b be the numbers of hours recorded in the other two questionnaires.		
Note that $\dfrac{a+b+19+20}{4}=18$.		
Therefore, we have $a+b=33$.	1M	
If the two medians are the same, then we have $a \le 16$ and $b \le 16$.	1M	
Hence, we have $a+b \le 32$.		
It is impossible since $a+b=33$.		
Thus, it is not possible that the two medians are the same.	1A	f.t.
Let a and b be the numbers of hours recorded in the other two questionnaires.		
Note that $\dfrac{a+b+19+20}{4}=18$.		
Therefore, we have $b=33-a$.	1M	
If the two medians are the same, then we have $a \le 16$ and $b \le 16$.	1M	
Hence, we have $a \le 16$ and $33-a \le 16$.		
So, we have $a \le 16$ and $a \ge 17$.		
It is impossible since $17 > 16$.		
Thus, it is not possible that the two medians are the same.	1A	f.t.
	----------(4)	
11. (a) Let $C=r+sA$, where r and s are non-zero constants.	1A	
So, we have $r+2s=62$ and $r+6s=74$.	1M	for either substitution
Solving, we have $r=56$ and $s=3$.	1A	for both correct
The required cost		
$= 56+3(13)$		
$= \$ 95$	1A	u−1 for missing unit
	----------(4)	
(b) Since the volume of the larger can is 8 times that of the can described in (a), the surface area of the larger can is 4 times that of the can described in (a).	1M	
The surface area of the larger can		
$= (13)(4)$		
$= 52 \text{ m}^2$		
The required cost		
$= 56+3(52)$		
$= \$ 212$	1A	u−1 for missing unit
	----------(2)	

Solution	Marks	Remarks
12. (a) The required volume		
$=\frac{1}{3}\pi(48)^2(96)$	1M	
$=73\,728\pi\ \text{cm}^3$	1A	u–1 for missing unit
	----------(2)	
(b) (i) The required volume		
$=\frac{2}{3}\pi(60)^3$	1M	
$=144\,000\pi\ \text{cm}^3$	1A	u–1 for missing unit
	----------(2)	
(ii) Let h cm be the height of the frustum under the surface of the milk and r cm be the base radius of the circular cone above the surface of the milk.		
h		
$=\sqrt{60^2-48^2}$	1M	
$=36$		
$\frac{r}{48}=\frac{96-36}{96}$		
$r=30$		
The volume of the milk remaining in the vessel		
$=144\,000\pi-\left(73\,728\pi-\frac{1}{3}\pi(30)^2(96-36)\right)$	1M	
$=88\,272\pi\ \text{cm}^3$		
$\approx 0.2773146667\ \text{m}^3$		
$<0.3\ \text{m}^3$		
Thus, the claim is disagreed.	1A	f.t.
The height of the frustum under the surface of the milk		
$=\sqrt{60^2-48^2}$	1M	
$=36$ cm		
The volume of the milk remaining in the vessel		
$=144\,000\pi-\left(73\,728\pi\left(1-\left(\frac{96-36}{96}\right)^3\right)\right)$	1M	
$=88\,272\pi\ \text{cm}^3$		
$\approx 0.2773146667\ \text{m}^3$		
$<0.3\ \text{m}^3$		
Thus, the claim is disagreed.	1A	f.t.
	----------(3)	

Solution	Marks	Remarks
13. (a) $k(2)^3-21(2)^2+24(2)-4=0$	1M	
$8k=40$		
$k=5$	1A	
	----------(2)	
(b) (i) The area of $OPQR$		
$=m(15m^2-63m+72)$	1A	
$=15m^3-63m^2+72m$		
(ii) Note that the area of $OPQR$ is 12 .		
$15m^3-63m^2+72m=12$	1M	
$5m^3-21m^2+24m-4=0$		
$(m-2)(5m^2-11m+2)=0$	1M+1A	1M for $(m-2)(am^2+bm+c)$
$(m-2)^2(5m-1)=0$		
$m=2$ or $m=\frac{1}{5}$		
So, there are only two different positions of Q such that the area of the rectangle $OPQR$ is 12 . Thus, there are no three different positions of Q such that the area of the rectangle $OPQR$ is 12 .	1A	f.t.
	----------(5)	
14. (a) (i) Γ is parallel to L .	1A	
(ii) Note that the y-intercept of Γ is -2 .	1A	
The slope of L		
$=\frac{-1-0}{0-3}$	1M	
$=\frac{1}{3}$	1A	
The equation of Γ is		
$y+2=\frac{1}{3}(x-0)$		
$x-3y-6=0$	1A	or equivalent
	----------(5)	
(b) (i) Note that the coordinates of Q are $(6,0)$.	1A	
Since $6-3(0)-6=0$, Γ passes through Q .	1A	f.t.
(ii) Note that both QH and QK are radii of the circle. Also note that both the heights of ΔAQH and ΔBQK are the distance between L and Γ .	1M	either one
Therefore, the area of ΔAQH is equal to the area of ΔBQK .		
Thus, the required ratio is 1 : 1 .	1A	
	----------(4)	

Solution	Marks	Remarks
15. (a) The standard deviation		
$=10(1+20\%)$		
$=12$ marks	1A	
	----------(1)	
(b) Let x be the test score and m be the mean of the test scores before the score adjustment.		
The standard score before the score adjustment		
$=\dfrac{x-m}{10}$		
The standard score after the score adjustment		
$=\dfrac{(x(1+20\%)+5)-(m(1+20\%)+5)}{12}$	1M	
$=\dfrac{1.2(x-m)}{12}$		
$=\dfrac{x-m}{10}$		
Thus, there is no change in the standard score of each student due to the score adjustment.	1A	f.t.
	----------(2)	
16. (a) The required probability		
$=\dfrac{(C_4^8)(C_1^2)^4}{C_4^{16}}$	1M	for either numerator or denominator
$=\dfrac{8}{13}$	1A	r.t 0.615
The required probability		
$=\left(\dfrac{16}{16}\right)\left(\dfrac{14}{15}\right)\left(\dfrac{12}{14}\right)\left(\dfrac{10}{13}\right)$	1M	for either numerator or denominator
$=\dfrac{8}{13}$	1A	r.t 0.615
	----------(2)	
(b) The required probability		
$=1-\dfrac{8}{13}$	1M	for $1-$(a)
$=\dfrac{5}{13}$	1A	r.t. 0.385
The required probability		
$=\dfrac{C_2^8}{C_4^{16}}+\dfrac{(C_1^8)(C_2^2)(C_2^7)(C_1^2)^2}{C_4^{16}}$	1M	for considering 2 cases
$=\dfrac{5}{13}$	1A	r.t 0.385
The required probability		
$=\dfrac{C_2^8}{C_4^{16}}+\dfrac{(C_2^8)(C_1^2)^2(C_1^6)(C_2^2)}{C_4^{16}}$	1M	for considering 2 cases
$=\dfrac{5}{13}$	1A	r.t 0.385
	----------(2)	

Solution	Marks	Remarks
17. (a) Note that the radius of C is 10 .	1M	can be absorbed
Thus, the equation of C is $(x-6)^2+(y-10)^2=10^2$.	1A	$x^2+y^2-12x-20y+36=0$
	----------(2)	
(b) The equation of L is $y=-x+k$.	1M	
Putting $y=-x+k$ in $x^2+y^2-12x-20y+36=0$,		
we have $x^2+(-x+k)^2-12x-20(-x+k)+36=0$.	1M	
So, we have $2x^2+(8-2k)x+(k^2-20k+36)=0$.		
The x-coordinate of the mid-point of AB		
$=\dfrac{\frac{-(8-2k)}{2}}{2}$	1M	for sum of roots
$=\dfrac{k-4}{2}$	1A	
The y-coordinate of the mid-point of AB		
$=\dfrac{-(k-4)}{2}+k$		
$=\dfrac{k+4}{2}$	1A	
Thus, the required coordinates are $\left(\dfrac{k-4}{2},\dfrac{k+4}{2}\right)$.		
The equation of L is $y=-x+k$.	1M	
Note that the equation of the straight line passing through the centre of C and perpendicular to L is $y-10=1(x-6)$.	1M	
Solving the system of linear equations $\begin{cases} y=-x+k \\ x-y+4=0 \end{cases}$,	1M	for solving
we have $\begin{cases} x=\dfrac{k-4}{2} \\ y=\dfrac{k+4}{2} \end{cases}$.	1A+1A	
Thus, the required coordinates are $\left(\dfrac{k-4}{2},\dfrac{k+4}{2}\right)$.		

只限教師參閱 FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
The equation of L is $y=-x+k$.	1M	
Putting $y=-x+k$ in $x^2+y^2-12x-20y+36=0$,		
we have $x^2+(-x+k)^2-12x-20(-x+k)+36=0$.	1M	
Hence, we have $2x^2+(8-2k)x+(k^2-20k+36)=0$.		
Note that $\sqrt{(8-2k)^2-4(2)(k^2-20k+36)}=2\sqrt{-k^2+32k-56}$.		
So, the x-coordinates of A and B are $\dfrac{k-4+\sqrt{-k^2+32k-56}}{2}$ and $\dfrac{k-4-\sqrt{-k^2+32k-56}}{2}$.	1M	for solving
The x-coordinate of the mid-point of AB $=\dfrac{\dfrac{k-4+\sqrt{-k^2+32k-56}}{2}+\dfrac{k-4-\sqrt{-k^2+32k-56}}{2}}{2}$		
$=\dfrac{k-4}{2}$	1A	
The y-coordinate of the mid-point of AB $=\dfrac{-(k-4)}{2}+k$		
$=\dfrac{k+4}{2}$	1A	
Thus, the required coordinates are $\left(\dfrac{k-4}{2},\dfrac{k+4}{2}\right)$.		
	----------(5)	

只限教師參閱 FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
18. (a) By sine formula, we have		
$\frac{AP}{\sin \angle PBA} = \frac{AB}{\sin \angle APB}$	1M	
$\frac{AP}{\sin 60°} = \frac{20}{\sin(180° - 60° - 72°)}$		
$AP \approx 23.30704256$ cm		
$AP \approx 23.3$ cm	1A	r.t. 23.3 cm
Thus, the length of AP is 23.3 cm .		
	----------(2)	
(b) (i) Let S be the foot of the perpendicular from P to AD .		
PS $= AP \sin \angle PAD$ $\approx 23.30704256 \sin 72°$ ≈ 22.1663147 cm AS $= AP \cos \angle PAD$ $\approx 23.30704256 \cos 72°$ ≈ 7.202272239 cm	1M	either one
By sine formula, we have		
$\frac{PB}{\sin \angle PAB} = \frac{AB}{\sin \angle APB}$		
$\frac{PB}{\sin 72°} = \frac{20}{\sin(180° - 60° - 72°)}$		
$PB \approx 25.59545552$ cm		
Let T be the foot of the perpendicular from P to BC .		
$PT^2 = PB^2 - AS^2$		
$PT^2 \approx (25.59545552)^2 - (7.202272239)^2$		
$PT \approx 24.56124219$ cm		
Note that $\alpha = \angle PTS$.	1M	
By cosine formula, we have		
$\cos\alpha = \frac{PT^2 + ST^2 - PS^2}{2(PT)(ST)}$	1M	
$\cos\alpha \approx \frac{(24.56124219)^2 + (20)^2 - (22.1663147)^2}{2(24.56124219)(20)}$		
$\alpha \approx 58.59703733°$		
$\alpha \approx 58.6°$	1A	r.t. 58.6°
(ii) Let X be the projection of P on the base $ABCD$.		
Then, we have $\beta = \angle PBX$.	1M	
Note that $PB > PT$.		
$\sin\alpha$ $= \frac{PX}{PT}$ $> \frac{PX}{PB}$ $= \sin \angle PBX$ $= \sin\beta$		
Since α and β are acute angles, α is greater than β .	1A	f.t.
	----------(6)	

只限教師參閱　　FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
19. (a) (i) Note that $\begin{cases} ab^2 = 254\,100 \\ ab^4 = 307\,461 \end{cases}$.	1M	
So, we have $b^2 = \dfrac{307\,461}{254\,100}$.		
Solving, we have $b = 1.1$ and $a = 210\,000$.	1A+1A	
The required weight $= (210\,000)(1.1^{(2)(4)})$ $= 450\,153.6501$ tonnes	1A	r.t. 450 000 tonnes
(ii) The total weight of the goods $= ab^2 + ab^4 + \cdots + ab^{2n}$ $= \dfrac{ab^2(b^{2n}-1)}{b^2-1}$	1M	
$= \dfrac{(210\,000)(1.1)^2((1.1)^{2n}-1)}{1.1^2-1}$ $= 1\,210\,000((1.1)^{2n}-1)$ tonnes	1A	
	----------(6)	
(b) (i) Note that $A(4) = 450\,153.65 > 420\,000 = 2a$. Also note that $(1.1)^{2m} > (1.1)^m$ for any positive integer m.		
$A(m+4)$	1M	for considering $A(m+4)$
$= (1.1)^{2m}A(4)$ $> (1.1)^{2m}(2a)$ $> (1.1)^m(2a)$ $= B(m)$		
Thus, the claim is agreed.	1A	f.t.
(ii) Let n be the number of years elapsed since the start of the operation of X. The total weight of the goods handled by Y $= 2ab + 2ab^2 + \cdots + 2ab^{n-4}$ $= \left(\dfrac{2ab(b^{n-4}-1)}{b-1}\right)$ tonnes, where $n > 4$		
$1\,210\,000((1.1)^{2n}-1) + \dfrac{420\,000(1.1)((1.1)^{n-4}-1)}{1.1-1} > 20\,000\,000$	1M+1A	
$121(1.1^{2n}) + 462(1.1^{n-4}) - 2583 > 0$		
$121(1.1^4)(1.1^n)^2 + 462(1.1^n) - 2583(1.1^4) > 0$	1M	
$1.1^n > 3.496831134$ or $1.1^n < -6.10470069$ (rejected)		
$n\log 1.1 > \log 3.496831134$	1M	
$n > 13.13455888$ Note that n is an integer.		
Thus, the new facilities should be installed in the 14th year since the start of the operation of X.	1A	
	----------(7)	

只限教師參閱 FOR TEACHERS' USE ONLY