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香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2022年香港中學文憑考試
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2022

數學　　必修部分　　試卷一

MATHEMATICS　COMPULSORY PART　PAPER 1

評卷參考
MARKING SCHEME

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Hong Kong Diploma of Secondary Education Examination
Mathematics Compulsory Part Paper 1

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

5. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Solution	Marks	Remarks
1. $\dfrac{(a^3b^{-2})^4}{a^{-5}b^6}$		
$=\dfrac{a^{12}b^{-8}}{a^{-5}b^6}$	1M	for $(m^h)^k = m^{hk}$ or $(mn)^l = m^l n^l$
$=\dfrac{a^{12+5}}{b^{6+8}}$	1M	for $\dfrac{x^p}{x^q} = x^{p-q}$ or $y^{-r} = \dfrac{1}{y^r}$
$=\dfrac{a^{17}}{b^{14}}$	1A	
	----------(3)	
2. Note that $x+y=456$ and $7x=y$.	1A	for either correct
So, we have $x+7x=456$.	1M	for getting a linear equation in x or y only
Solving, we have $x=57$.	1A	
x $=\dfrac{456}{1+7}$	1M+1A	1M for fraction
$=57$	1A	
	----------(3)	
3. $\dfrac{3}{k-9}+\dfrac{2}{5k+6}$		
$=\dfrac{3(5k+6)+2(k-9)}{(k-9)(5k+6)}$	1M	
$=\dfrac{15k+18+2k-18}{(k-9)(5k+6)}$	1M	
$=\dfrac{17k}{(k-9)(5k+6)}$	1A	or equivalent
	----------(3)	
4. (a) $9c^2-6c+1$		
$=(3c-1)^2$	1A	or equivalent
(b) $(4c+d)^2-9c^2+6c-1$		
$=(4c+d)^2-(9c^2-6c+1)$		
$=(4c+d)^2-(3c-1)^2$	1M	for using the result of (a)
$=(4c+d+3c-1)(4c+d-(3c-1))$	1M	
$=(7c+d-1)(c+d+1)$	1A	or equivalent
	----------(4)	

Solution	Marks	Remarks
5. Let $\$ x$ be the cost of the fan.		
$(26\%)x = 78$	1M	
$x = \dfrac{78}{0.26}$		
$x = 300$		
The selling price of the fan		
$= 300 + 78$	1M	
$= \$ 378$		
Let $\$ y$ be the marked price of the fan.		
$(70\%)y = 378$	1M	
$y = \dfrac{378}{0.7}$		
$y = 540$	1A	
Thus, the marked price of the fan is $\$ 540$.		
The marked price of the fan		
$= \dfrac{(78)(1+26\%)}{(26\%)(70\%)}$	1M+1M+1M	1M for fraction + 1M for numerator + 1M for denominator
$= \$ 540$	1A	
	----------(4)	
6. (a) $-2(3x+2) > x+10$		
$-6x-4 > x+10$		
$-6x-x > 10+4$	1M	for putting x on one side
$-7x > 14$		
$x < -2$	1A	
$2x \leq -8$		
$x \leq -4$		
Therefore, we have $x < -2$ or $x \leq -4$.		
Thus, the solution of (*) is $x < -2$.	1A	
(b) -3	1A	
	----------(4)	
7. (a) The coordinates of S' are $(5, 12)$.	1A	
The coordinates of T' are $(-3, 7)$.	1A	
(b) The slope of $S'T'$		
$= \dfrac{12-7}{5-(-3)}$	1M	
$= \dfrac{5}{8}$	1A	0.625
	----------(4)	

Solution		Marks	Remarks
8. (a) $\angle ACB = \angle CAD$	(alt. $\angle$s, $AD // BC$)		[(內)錯角，$AD // BC$]
$\angle CAD = \angle ADE$	(alt. $\angle$s, $AC // ED$)		[(內)錯角，$AC // ED$]
$\angle ACB = \angle ADE$			
$\angle ABC = \angle AED$	(given)		[已知]
$AB = AE$	(given)		[已知]
$\Delta ABC \cong \Delta AED$	(AAS)		
Marking Scheme:			
Case 1 Any correct proof with correct reasons.		2	
Case 2 Any correct proof without reasons.		1	
(b) $\angle BAC$			
$= \angle DAE$		1M	
$= 87°$			
$\angle ACB$			
$= 180° - \angle BAC - \angle ABC$			
$= 180° - 87° - 39°$			
$= 54°$			
$\angle CAD$			
$= \angle ACB$			
$= 54°$			
Note that $AC = AD$.			
$\angle ACD$			
$= \angle ADC$			
$= \dfrac{180° - \angle CAD}{2}$		1M	
$= \dfrac{180° - 54°}{2}$			
$= 63°$		1A	
		----------(5)	
9. (a) 12		1A	
(b) Note that $a = 3$ and $b = 5$.			
The mean			
$= \dfrac{12(3)+17(9)+22(5)+27(3)}{20}$		1M	
$= 19$ minutes		1A	
(c) The required probability			
$= \dfrac{12}{20}$		1M	for numerator
$= \dfrac{3}{5}$		1A	0.6
		----------(5)	

Solution	Marks	Remarks
10. (a) Let $f(x) = ax^2 + bx$, where a and b are non-zero constants.	1A	
So, we have $16a + 4b = 96$ and $25a - 5b = 15$.	1M	for either substitution
Solving, we have $a = 3$ and $b = 12$.	1A	for both correct
Thus, we have $f(x) = 3x^2 + 12x$.		
	----------(3)	
(b) The x-intercepts of the graph of $y = 8f(x)$ are 0 and -4 .	1A	for both correct
	----------(1)	
(c) The equation $3x^2 + 12x - k = 0$ has two distinct real roots.		
$12^2 - 4(3)(-k) > 0$	1M	
$k > -12$	1A	
	----------(2)	
11. (a) $36 - (20 + a) = 14$	1M	
$a = 2$	1A	
$30 + b = 31$		
$b = 1$	1A	
	----------(3)	
(b) (i) The original mode $= 36$	1M	either one
The new mode $= 36$		
Thus, there is no change in the mode of the distribution.	1A	f.t.
(ii) There are two cases.		
Case 1: The player of age 17 leaves the football team. The standard deviation of the distribution ≈ 7.162537194	1M	either one
Case 2: The player of age 43 leaves the football team. The standard deviation of the distribution ≈ 7.132307207		
Thus, the greatest possible standard deviation of the distribution is 7.16 .	1A	f.t.
	----------(4)	

Solution	Marks	Remarks
12. (a) The coordinates of G		
$=\left(\dfrac{154}{2},\dfrac{128}{2}\right)$	1M	
$=(77,64)$		
The distance between G and H		
$=\sqrt{(77-65)^2+(64-48)^2}$	1M	
$=20$	1A	
	----------(3)	
(b) (i) GH is perpendicular to GP.	1M	
(ii) The radius of C		
$=\sqrt{\left(\dfrac{154}{2}\right)^2+\left(\dfrac{128}{2}\right)^2-224}$	1M	
$=99$		
$HP^2=GH^2+GP^2$	1M	
$HP^2=20^2+99^2$		
$HP=101$		
The perimeter of ΔGHP		
$=GH+GP+HP$		
$=20+99+101$		
$=220$	1A	
	----------(4)	

Solution	Marks	Remarks
13. (a) The ratio of the volume of the smaller sphere to the volume of the larger sphere is $8 : 27$.	1M	
The volume of the smaller sphere		
$= \frac{4}{3}\pi(9)^3\left(\frac{8}{27}\right)$	1M	
$= 288\pi \text{ cm}^3$	1A	
	----------(3)	
(b) The volume of A		
$= \frac{1}{3}\pi(6^2)(10)$	1M	
$= 120\pi \text{ cm}^3$		
The volume of B		
$= 288\pi + \frac{4}{3}\pi(9)^3 - 120\pi$	1M	
$= 1140\pi \text{ cm}^3$		
$\frac{\text{The volume of } B}{\text{The volume of } A}$		
$= \frac{1140\pi}{120\pi}$	1M	either one
$= \frac{19}{2}$		
$\left(\frac{\text{The base radius of } B}{\text{The base radius of } A}\right)^3$		
$= \left(\frac{12}{6}\right)^3$		
$= 8$		
$\neq \frac{19}{2}$		
So, A and B are not similar.		
Thus, the claim is not correct.	1A	f.t.
	----------(4)	

Solution	Marks	Remarks
14. (a) Let $p(x) = (mx+n)(x^2-2x+3)+x+13$, where m and n are constants.	1M	
Therefore, we have $p(x) = mx^3+(n-2m)x^2+(3m-2n+1)x+3n+13$.		
So, we have $m=2$ and $3n+13=-20$.	1M	for either one
Solving, we have $m=2$ and $n=-11$.		
Hence, we have $p(x)=2x^3-15x^2+29x-20$.		
Thus, we have $a=-15$ and $b=29$.	1A	for both correct
	----------(3)	
(b) $p(5)$	1M	
$=2(5)^3-15(5)^2+29(5)-20$		
$=0$		
Thus, $x-5$ is a factor of $p(x)$.	1A	f.t.
	----------(2)	
(c) $p(x)=0$		
$(x-5)(2x^2-5x+4)=0$	1M	
$x-5=0$ or $2x^2-5x+4=0$		
$(-5)^2-4(2)(4)$	1M	
$=-7$		
<0		
So, the quadratic equation $2x^2-5x+4=0$ has no real roots.		
Hence, the quadratic equation $2x^2-5x+4=0$ has no irrational roots.		
Note that 5 is not an irrational number.		
Therefore, the equation $p(x)=0$ has no irrational roots.		
Thus, the claim is disagreed.	1A	f.t.
	----------(3)	

Solution	Marks	Remarks
15. (a) The required probability		
$=\dfrac{C_2^{10}C_2^{12}}{C_4^{22}}$	1M	for numerator
$=\dfrac{54}{133}$	1A	r.t. 0.406
The required probability		
$=6\left(\dfrac{10}{22}\right)\left(\dfrac{9}{21}\right)\left(\dfrac{12}{20}\right)\left(\dfrac{11}{19}\right)$	1M	for $6p_1p_2p_3p_4$
$=\dfrac{54}{133}$	1A	r.t. 0.406
	----------(2)	
(b) The required probability		
$=1-\dfrac{54}{133}$	1M	for $1-(a)$
$=\dfrac{79}{133}$	1A	r.t. 0.594
The required probability		
$=\dfrac{C_4^{10}}{C_4^{22}}+\dfrac{C_3^{10}C_1^{12}}{C_4^{22}}+\dfrac{C_1^{10}C_3^{12}}{C_4^{22}}+\dfrac{C_4^{12}}{C_4^{22}}$	1M	for $p_5+p_6+p_7+p_8$
$=\dfrac{79}{133}$	1A	r.t. 0.594
	----------(2)	
16. (a) $g(x)$		
$=3x^2+12kx+16k^2+8$		
$=3(x^2+4kx)+16k^2+8$		
$=3(x^2+4kx+4k^2)+4k^2+8$	1M	
$=3(x+2k)^2+4k^2+8$		
Thus, the coordinates of the vertex are $(-2k\,,\,4k^2+8)$.	1A	
	----------(2)	
(b) The coordinates of B are $(2k\,,\,8k^2+16)$.	1M	
Note that $AM:MB=1:3$.		
The coordinates of M		
$=\left(\dfrac{3(-2k)+(2k)}{1+3},\dfrac{3(4k^2+8)+(8k^2+16)}{1+3}\right)$	1M	
$=(-k\,,\,5k^2+10)$	1A	
	----------(3)	

Solution	Marks	Remarks
17. (a) Note that $\alpha+\beta=-c$ and $\alpha\beta=-9$.	1M	for either one
$\alpha^2+\beta^2$		
$=(\alpha+\beta)^2-2\alpha\beta$	1M	
$=(-c)^2-2(-9)$		
$=c^2+18$	1A	
	----------(3)	
(b) $\alpha^2+\beta^2-c^2=85-(\alpha^2+\beta^2)$	1M	
$c^2+18-c^2=85-(c^2+18)$		
$c^2=49$		
Note that the 1st term and the common difference of the sequence are 49 and 18 respectively.		
$\frac{n}{2}(2(49)+18(n-1))>2\times10^6$	1M	
$9n^2+40n-2\times10^6>0$		
$n<\frac{-40-\sqrt{40^2-4(9)(-2\times10^6)}}{2(9)}$ or $n>\frac{-40+\sqrt{40^2-4(9)(-2\times10^6)}}{2(9)}$	1M	
$n<-473.6319808$ or $n>469.1875364$		
Thus, the least value of n is 470 .	1A	
	----------(4)	

Solution	Marks	Remarks
18. (a) (i) $QR^2 = PQ^2 + PR^2 - 2(PQ)(PR)\cos\angle QPR$	1M	
$QR^2 = 30^2 + 25^2 - 2(30)(25)\cos 95°$		
$QR \approx 40.69070673$		
$QR \approx 40.7$ cm	1A	r.t. 40.7 cm
Thus, the length of QR is 40.7 cm .		
(ii) $\dfrac{\sin\angle PQR}{PR} = \dfrac{\sin\angle QPR}{QR}$	1M	
$\dfrac{\sin\angle PQR}{25} \approx \dfrac{\sin 95°}{40.69070673}$		
$\angle PQR \approx 37.73809375°$ or $\angle PQR \approx 142.2619063°$ (rejected)		
Thus, we have $\angle PQR \approx 37.7°$.	1A	r.t. 37.7°
	----------(4)	
(b) $PM^2 = PQ^2 + QM^2 - 2(PQ)(QM)\cos\angle PQR$		
$PM^2 \approx 30^2 + \left(\dfrac{40.69070673}{2}\right)^2 - 2(30)\left(\dfrac{40.69070673}{2}\right)\cos 37.73809375°$		
$PM \approx 18.66993831$ cm		
Let D and N be the projections of R and M on the horizontal ground respectively.		
$MN = \dfrac{1}{2}RD = \dfrac{1}{2}PR\sin 70° = \dfrac{1}{2}(25)\sin 70° \approx 11.74615776$ cm	1M	
Note that the angle between PM and the horizontal ground is $\angle MPN$.	1M	
$\sin\angle MPN = \dfrac{MN}{PM}$		
$\sin\angle MPN \approx \dfrac{11.74615776}{18.66993831}$		
$\angle MPN \approx 38.98730493°$		
$\angle MPN < 40°$		
Thus, the claim is not correct.	1A	f.t.
	----------(3)	

Solution	Marks	Remarks
19. (a) The slope of AG		
$=\frac{112-12}{83-158}$		
$=\frac{-4}{3}$		
The required equation is		
$y-12=\frac{-4}{3}(x-158)$	1M	
$4x+3y-668=0$	1A	or equivalent
	----------(2)	
(b) The radius of C is 75 .		
Since $83+75=158$, AP is vertical or AQ is vertical.		
So, the coordinates of P or the coordinates of Q are $(158,112)$.	1M	
Note that $\Delta AGP \cong \Delta AGQ$ and $AG \perp PQ$.		
Hence, the slope of PQ is $\frac{3}{4}$.		
The equation of PQ is $y-112=\frac{3}{4}(x-158)$.		
Solving $y-112=\frac{3}{4}(x-158)$ and $4x+3y-668=0$, we have	1M	
$x=110$ and $y=76$.		
Thus, the coordinates of the point of intersection of AG and PQ are $(110, 76)$.	1A	
	----------(3)	
(c) Let I and r be the centre and the radius of the inscribed circle of ΔAPQ respectively.		
Note that $AP=AQ$ and I lies on AG .		
The x-coordiante of I		
$=158-r$	1M	accept $110+\frac{3r}{5}$
The y-coordinate of I		
$=\frac{-4}{3}(158-r)+\frac{668}{3}$		
$=\frac{4r}{3}+12$		
So, the coordinates of I are $\left(158-r,\frac{4r}{3}+12\right)$.		
The distance between I and the point of intersection of AG and PQ is r .		
$((158-r)-110)^2+\left(\left(\frac{4r}{3}+12\right)-76\right)^2=r^2$	1M	accept $158-r=110+\frac{3r}{5}$
$\frac{16}{9}r^2-\frac{800}{3}r+6\,400=0$		
$r^2-150r+3\,600=0$		
$r=30$ or $r=120$ (rejected)	1M	
Therefore, we have $r=30$.		
Hence, the coordinates of I are $(128,52)$.		
Thus, the required equation is $(x-128)^2+(y-52)^2=30^2$.	1A	$x^2+y^2-256x-104y+18\,188=0$
	----------(4)	

Solution	Marks	Remarks
(d) Note that $\angle APG = \angle AQG = 90°$ and $\angle APG + \angle AQG = 180°$.		
So, $APGQ$ is a cyclic quadrilateral and AG is a diameter of the circumcircle of ΔAPQ .		
The radius of the circumcircle of ΔAPQ		
$=\frac{1}{2}\sqrt{(83-158)^2+(112-12)^2}$	1M	
$=\frac{125}{2}$		
By (c), the radius of the inscribed circle of ΔAPQ is 30 .		
The ratio of the area of the inscribed circle to the area of the circumcircle of ΔAPQ		
$=30^2:\left(\frac{125}{2}\right)^2$	1M	
$=144:625$		
$\neq 1:4$		
Thus, the claim is disagreed.	1A	f.t.
	----------(3)	