香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2012年香港中學文憑考試
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2012

數學 延伸部分
單元一（微積分與統計）

MATHEMATICS Extended Part
Module 1 (Calculus and Statistics)

評卷參考

MARKING SCHEME

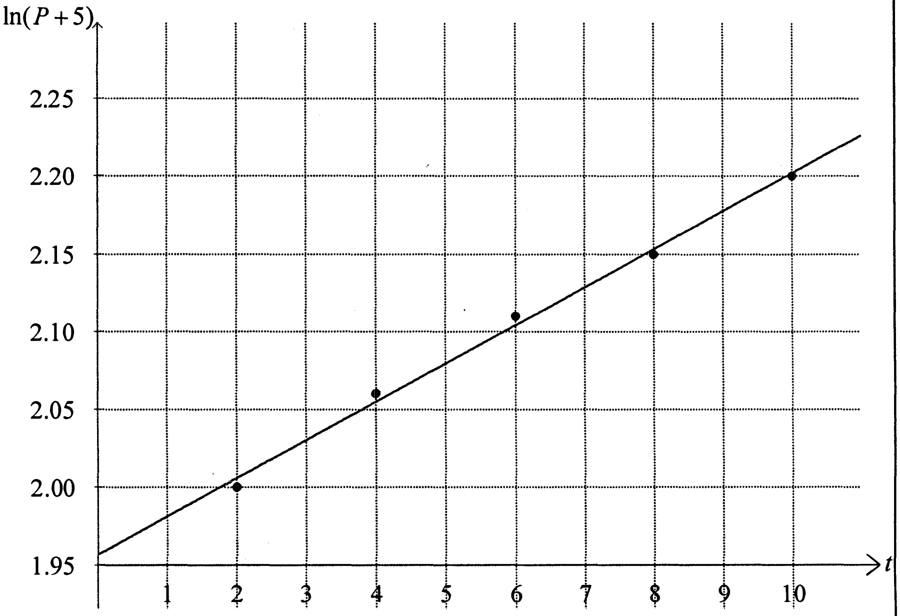
本評卷參考乃香港考試及評核局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

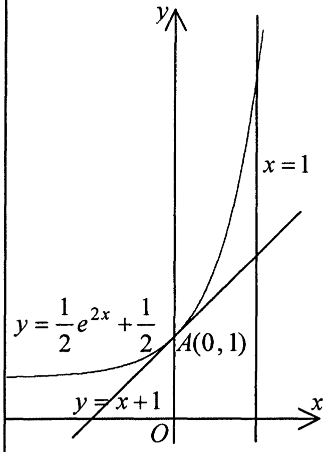
This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

Solution	Marks	Remarks
1. (a) $(1+3x)^n = 1 + C_1^n(3x) + C_2^n(3x)^2 + \cdots$		
$= 1+3nx+\frac{9n(n-1)}{2}x^2+\cdots$	1A	
(b) $e^{-2x}(1+3x)^n = \left[1+(-2x)+\frac{(-2x)^2}{2!}+\cdots\right]\left[1+3nx+\frac{9n(n-1)}{2}x^2+\cdots\right]$	1A	For $1+(-2x)+\frac{(-2x)^2}{2!}+\cdots$
$=(1-2x+2x^2+\cdots)\left[1+3nx+\frac{9n(n-1)}{2}x^2+\cdots\right]$		
$\therefore\ 1\cdot\frac{9n(n-1)}{2}+(-2)(3n)+2\cdot 1=62$	1M	
$9n^2-21n-120=0$		
$n=5\ \boxed{\text{or } \frac{-8}{3} \text{ (rejected)}}$	1A	
	(4)	
2. Let $u=4t+1$.	1M	
$\mathrm{d}u=4\mathrm{d}t$		
When $t=0$, $u=1$; when $t=2$, $u=9$.		
The change in the value of the flat		
$=\int_0^2 \frac{t}{\sqrt{4t+1}}\mathrm{d}t$	1M	
$=\int_1^9 \frac{1}{\sqrt{u}}\cdot\frac{u-1}{4}\frac{\mathrm{d}u}{4}$		
$=\frac{1}{16}\int_1^9\left(\sqrt{u}-\frac{1}{\sqrt{u}}\right)\mathrm{d}u$		
$=\frac{1}{16}\left[\frac{2}{3}u^{\frac{3}{2}}-2u^{\frac{1}{2}}\right]_1^9$	1A	For $\frac{1}{16}\left[\frac{2}{3}u^{\frac{3}{2}}-2u^{\frac{1}{2}}\right]$
$=\frac{5}{6}$	1A	
Hence the percentage change $=\frac{\frac{5}{6}}{3}\times 100\%$		
$=27\frac{7}{9}\%$	1A	OR 27.7778%
	(5)	
3. (a) $P=ae^{\frac{kt}{40}}-5$		
$\ln(P+5)=\frac{k}{40}t+\ln a$	1A	
(b)		

t	2	4	6	8	10
P	2.36	2.81	3.23	3.55	4.01
$\ln(P+5)$	2.00	2.06	2.11	2.15	2.20

Solution	Marks	Remarks
(table above)	1M	
From the graph on the next page, $\ln a \approx 1.96$	1M	
$a\approx 7$		Either one
$\frac{k}{40}\approx\frac{2.21-1.96}{10-0}$		
$k\approx 1$	1A	For both a and k

Solution	Marks	Remarks
	1A	
	(5)	
4. (a) $y=\sqrt[3]{\dfrac{3x-1}{x-2}}$		
$\ln y=\dfrac{1}{3}\ln(3x-1)-\dfrac{1}{3}\ln(x-2)$	1A	
$\therefore\ \dfrac{1}{y}\cdot\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{3x-1}-\dfrac{1}{3(x-2)}$	1A	
(b) By (a), $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\left[\dfrac{1}{3x-1}-\dfrac{1}{3(x-2)}\right]\sqrt[3]{\dfrac{3x-1}{x-2}}$	1A	
$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\dfrac{1}{3x-1}-\dfrac{1}{3(x-2)}\right]\sqrt[3]{\dfrac{3x-1}{x-2}}+\left[\dfrac{1}{3x-1}-\dfrac{1}{3(x-2)}\right]\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt[3]{\dfrac{3x-1}{x-2}}\right)$		
$=\left[\dfrac{-3}{(3x-1)^2}+\dfrac{1}{3(x-2)^2}\right]\sqrt[3]{\dfrac{3x-1}{x-2}}+\left[\dfrac{1}{3x-1}-\dfrac{1}{3(x-2)}\right]^2\sqrt[3]{\dfrac{3x-1}{x-2}}$ by (a)	1M	For using (a)
When $x=3$, $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\left\{\dfrac{-3}{(3\cdot 3-1)^2}+\dfrac{1}{3(3-2)^2}+\left[\dfrac{1}{3\cdot 3-1}-\dfrac{1}{3(3-2)}\right]^2\right\}\sqrt[3]{\dfrac{3\cdot 3-1}{3-2}}$	1M	
Alternative Solution		
When $x=3$, $y=2$ and so $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-5}{12}$.	1A	For both y and $\dfrac{\mathrm{d}y}{\mathrm{d}x}$
By (a), $\dfrac{1}{y}\cdot\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-\dfrac{1}{y^2}\cdot\dfrac{\mathrm{d}y}{\mathrm{d}x}\cdot\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-3}{(3x-1)^2}+\dfrac{1}{3(x-2)^2}$	1M	For chain rule
When $x=3$, $\dfrac{1}{2}\cdot\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-\dfrac{1}{2^2}\cdot\dfrac{-5}{12}\cdot\dfrac{-5}{12}=\dfrac{-3}{(3\cdot 3-1)^2}+\dfrac{1}{3(3-2)^2}$	1M	
i.e. $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{95}{144}$	1A	OR 0.6597
	(6)	

Solution	Marks	Remarks
5. (a) $\frac{dy}{dx}=e^{2x}$		
$y=\frac{1}{2}e^{2x}+C$	1A	
Since $A(0,1)$ lies on S, we have $1=\frac{1}{2}e^{2(0)}+C$.	1M	
i.e. $C=\frac{1}{2}$		
Hence the equation of S is $y=\frac{1}{2}e^{2x}+\frac{1}{2}$.	1A	
(b) At $A(0,1)$, $\frac{dy}{dx}=e^{2(0)}=1$.		
Hence the equation of L is $y-1=1(x-0)$.	1M	
i.e. $y=x+1$	1A	
(c) The area of the region bounded by S, L and the line $x=1$		
$=\int_0^1\left[\left(\frac{1}{2}e^{2x}+\frac{1}{2}\right)-(x+1)\right]dx$	1M	1M for $A=\int_0^1(y_1-y_2)dx$
$=\left[\frac{1}{4}e^{2x}-\frac{1}{2}x^2-\frac{1}{2}x\right]_0^1$		
$=\frac{e^2-5}{4}$	1A	OR 0.5973
	(7)	
6. (a) Let X be the weight of a student. The sample mean $\overline{X}\sim N\left(67,\frac{15^2}{36}\right)$.		
$P(\overline{X}>70)=P\left(Z>\frac{70-67}{\frac{15}{6}}\right)$	1M	
$=P(Z>1.2)$		
≈ 0.1151	1A	
(b) The sample proportion is $\frac{9}{36}=0.25$.	1A	
An approximate 95% confidence interval for the proportion		
$\approx\left(0.25-1.96\times\sqrt{\frac{0.25\times0.75}{36}},0.25+1.96\times\sqrt{\frac{0.25\times0.75}{36}}\right)$	1M	
$\approx(0.1085,0.3915)$	1A	
	(5)	
7. (a) $\frac{e^{-\lambda}}{0!}=0.1653$		
$\lambda=-\ln 0.1653$		
≈ 1.8	1A	
(b) P(no. of goals in a match <3) $=\frac{e^{-1.8}}{0!}+\frac{e^{-1.8}(1.8)}{1!}+\frac{e^{-1.8}(1.8)^2}{2!}$	1M	
≈ 0.7306	1A	

Solution	Marks	Remarks
(c) The number of goals scored in two matches by the team $\sim \text{Po}(3.6)$.		
$\therefore$ P(no. of goals in two matches < 3)		
$= \frac{e^{-3.6}}{0!} + \frac{e^{-3.6}(3.6)}{1!} + \frac{e^{-3.6}(3.6)^2}{2!}$	1M	
Alternative Solution P(no. of goals in two matches < 3) $= \text{P}(0, 0) + \text{P}(0, 1) + \text{P}(1, 0) + \text{P}(1, 1) + \text{P}(0, 2) + \text{P}(2\ ,0)$ $= \left(\frac{e^{-1.8}}{0!}\right)^2 + 2\left(\frac{e^{-1.8}}{0!}\right)\left[\frac{e^{-1.8}(1.8)}{1!}\right] + \left[\frac{e^{-1.8}(1.8)}{1!}\right]^2 + 2\left(\frac{e^{-1.8}}{0!}\right)\left[\frac{e^{-1.8}(1.8)^2}{2!}\right]$	1M	
≈ 0.3027	1A	
	(5)	
8. (a) $\text{P}(X = 1) + \text{P}(X = 3) + \cdots + \text{P}(X = 13) = 1$		
$0.1 + a + 0.25 + 0.15 + b + 0.05 = 1$	1M	
$a + b = 0.45$ -- (1)		
$\text{E}(X) = 5.5$		
$1 \times 0.1 + 3a + 4 \times 0.25 + 6 \times 0.15 + 9b + 13 \times 0.05 = 5.5$	1M	
$a + 3b = 0.95$ -- (2)		
Solving (1) and (2), we get $a = 0.2$ and $b = 0.25$.	1A	For both
(b) (i) $\text{P}(F \cap G) = 0.25 + 0.15$ $= 0.4$	1A	
(ii) $\text{P}(F) \times \text{P}(G) = (0.25 + 0.15 + 0.25 + 0.05)(0.1 + 0.2 + 0.25 + 0.15)$ $= 0.49$ $\neq \text{P}(F \cap G)$	1A	
Alternative Solution 1 $\text{P}(F \mid G) = \frac{\text{P}(F \cap G)}{\text{P}(G)}$ $= \frac{0.4}{0.1 + 0.2 + 0.25 + 0.15}$ ≈ 0.571428571 $\text{P}(F) = 0.25 + 0.15 + 0.25 + 0.05$ $= 0.7$ $\neq \text{P}(F \mid G)$	1A	
Alternative Solution 2 $\text{P}(G \mid F) = \frac{\text{P}(F \cap G)}{\text{P}(F)}$ $= \frac{0.4}{0.25 + 0.15 + 0.25 + 0.05}$ ≈ 0.571428571 $\text{P}(G) = 0.1 + 0.2 + 0.25 + 0.15$ $= 0.7$ $\neq \text{P}(G \mid F)$	1A	
Hence, F and G are not independent.	1	
	(6)	

Solution	Marks	Remarks
9. (a) Let X be the score of a student who had revised.		
$P(X \geq 43) = P\left(Z \geq \frac{43-59}{10}\right)$		
$= P(Z \geq -1.6)$		
≈ 0.9452	1A	
Let Y be the score of a student who had not revised.		
$P(Y \geq 43) = P\left(Z \geq \frac{43-35.2}{12}\right)$		Either one
$= P(Z \geq 0.65)$		
≈ 0.2578		
$\therefore$ P(pass the test) $\approx 0.73 \times 0.9452 + 0.27 \times 0.2578$	1M	
$= 0.759602$	1A	OR 0.7596
(b) P(a student had not revised for the test $\mid$ he passed the test)		
$= \frac{0.27 \times 0.2578}{0.759602}$	1M	
≈ 0.091634829		
≈ 0.0916	1A	
(c) P(4 students had not revised for the test among 10 passed students)		
$\approx C_6^{10}(0.091634829)^4(1-0.091634829)^6$	1M	
≈ 0.0083	1A	
	(7)	
10. (a) (i) $I = \int_1^4 \frac{1}{\sqrt{t}} e^{\frac{-t}{2}} \mathrm{d}t$		
$= \frac{1}{2} \cdot \frac{4-1}{6}\left[\frac{1}{\sqrt{1}}e^{\frac{-1}{2}} + \frac{1}{\sqrt{4}}e^{\frac{-4}{2}} + 2\left(\frac{1}{\sqrt{1.5}}e^{\frac{-1.5}{2}} + \frac{1}{\sqrt{2}}e^{\frac{-2}{2}} + \frac{1}{\sqrt{2.5}}e^{\frac{-2.5}{2}} + \frac{1}{\sqrt{3}}e^{\frac{-3}{2}} + \frac{1}{\sqrt{3.5}}e^{\frac{-3.5}{2}}\right)\right]$	1M	
≈ 0.692913377		
≈ 0.6929	1A	
(ii) $\frac{\mathrm{d}}{\mathrm{d}t}\left(t^{\frac{-1}{2}} e^{\frac{-t}{2}}\right) = \frac{-1}{2} t^{\frac{-3}{2}} e^{\frac{-t}{2}} + t^{\frac{-1}{2}} \cdot \frac{-1}{2} e^{\frac{-t}{2}}$	1M+1A	
$= \frac{-1}{2} e^{\frac{-t}{2}}\left(t^{\frac{-3}{2}} + t^{\frac{-1}{2}}\right)$		
$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(t^{\frac{-1}{2}} e^{\frac{-t}{2}}\right) = \frac{-1}{2}\left[e^{\frac{-t}{2}}\left(\frac{-3}{2} t^{\frac{-5}{2}} + \frac{-1}{2} t^{\frac{-3}{2}}\right) + \frac{-1}{2} e^{\frac{-t}{2}}\left(t^{\frac{-3}{2}} + t^{\frac{-1}{2}}\right)\right]$	1M+1A	
$= \frac{1}{4} e^{\frac{-t}{2}}\left(3t^{\frac{-5}{2}} + 2t^{\frac{-3}{2}} + t^{\frac{-1}{2}}\right)$		
> 0 for $1 \leq t \leq 4$.		
Hence the estimation in (i) is an over-estimate.	1	
	(7)	

Solution	Marks	Remarks
(b) Let $t = x^2$.	1M	
$\mathrm{d}t = 2x\mathrm{d}x$	$\}$ 1A	
When $t = 1$, $x = 1$; when $t = 4$, $x = 2$.		
$I = \int_1^4 \frac{1}{\sqrt{t}} e^{\frac{-t}{2}} \mathrm{d}t$		
$= \int_1^2 \frac{1}{x} e^{\frac{-x^2}{2}} 2x\mathrm{d}x$		
$= 2\int_1^2 e^{\frac{-x^2}{2}} \mathrm{d}x$	1	
	(3)	
(c) $2\int_1^2 e^{\frac{-x^2}{2}} \mathrm{d}x < 0.692913377$	1M	
$2\sqrt{2\pi} \int_1^2 \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \mathrm{d}x < 0.692913377$		
$2\sqrt{2\pi}(0.4772 - 0.3413) < 0.692913377$	1A	For 0.4772 and 0.3413
$\pi < 3.249593152$		
$\therefore \quad \pi < 3.25$	1	
	(3)	
11. (a) When $t = 35$, the intensity increased to a maximum and therefore $\frac{\mathrm{d}R}{\mathrm{d}t} = 0$.		
$\frac{a(30-35)+10}{(35-35)^2+b} = 0$	1A	
$a = 2$	1A	
	(2)	
(b) $\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{2(30-t)+10}{(t-35)^2+b}$		
Let $u = (t-35)^2 + b$.	1M	
$\mathrm{d}u = 2(t-35)\mathrm{d}t$		
$R = \int \frac{-2t+70}{(t-35)^2+b} \mathrm{d}t$		
$= \int \frac{-2t+70}{u} \frac{\mathrm{d}u}{2(t-35)}$		
$= -\ln\lvert u \rvert + C$		
$= -\ln[(t-35)^2 + b] + C$	1A	
$R\big\rvert_{t=T} = R\big\rvert_{t=0}$		
$-\ln[(T-35)^2 + b] + C = -\ln[(0-35)^2 + b] + C$	1M	
$(T-35)^2 = 35^2$		
$T = 70 \ \boxed{\text{or } 0 \text{ (rejected)}}$	1A	
	(4)	

<table>
<tr><th>Solution</th><th>Marks</th><th>Remarks</th></tr>
<tr><td>(c) $R\big|_{t=40}-R\big|_{t=41}=\ln\frac{61}{50}$</td><td></td><td></td></tr>
<tr><td>$-\ln[(40-35)^2+b]+C-\{-\ln[(41-35)^2+b]+C\}=\ln\frac{61}{50}$</td><td>1M</td><td></td></tr>
<tr><td>$-\ln(25+b)+\ln(36+b)=\ln\frac{61}{50}$</td><td></td><td></td></tr>
<tr><td>$\ln\frac{36+b}{25+b}=\ln\frac{61}{50}$</td><td></td><td></td></tr>
<tr><td>$b=25$</td><td>1A</td><td></td></tr>
<tr><td>$\therefore\quad R=-\ln[(t-35)^2+25]+C$</td><td></td><td></td></tr>
<tr><td>$R\big|_{t=35}=6$</td><td></td><td></td></tr>
<tr><td>$-\ln[(35-35)^2+25]+C=6$</td><td>1M</td><td></td></tr>
<tr><td>$C=6+\ln 25$</td><td></td><td></td></tr>
<tr><td>i.e. $R=-\ln[(t-35)^2+25]+6+\ln 25$</td><td>1A</td><td></td></tr>
<tr><td></td><td>(4)</td><td></td></tr>
<tr><td>(d) $\frac{dR}{dt}=\frac{2(30-t)+10}{(t-35)^2+25}$
$=\frac{70-2t}{t^2-70t+1250}$</td><td></td><td></td></tr>
<tr><td>$\frac{d^2R}{dt^2}=\frac{(t^2-70t+1250)(-2)-(70-2t)(2t-70)}{(t^2-70t+1250)^2}$</td><td>1M+1A</td><td></td></tr>
<tr><td>$=\frac{2t^2-140t+2400}{(t^2-70t+1250)^2}$</td><td></td><td></td></tr>
<tr><td>When the rate of change of the radiation intensity attains its greatest value, $\frac{d^2R}{dt^2}=0$.</td><td></td><td></td></tr>
<tr><td>$2t^2-140t+2400=0$
$t=30$ $\boxed{\text{or }40\text{ (rejected)}}$</td><td></td><td></td></tr>
<tr><td><table>
<tr><td>t</td><td>$0\le t<30$</td><td>$t=30$</td><td>$30<t\le 35$</td></tr>
<tr><td>$\frac{d^2R}{dt^2}$</td><td>+ve</td><td>0</td><td>-ve</td></tr>
</table></td><td>1M</td><td></td></tr>
<tr><td>Hence, the rate of change of the radiation intensity would attain its greatest value when $t=30$.</td><td>1A</td><td></td></tr>
<tr><td></td><td>(4)</td><td></td></tr>
</table>

Solution	Marks	Remarks
12. (a) (i) The sample mean $=\frac{56+\cdots+50}{16}$		
$=51.5625$	1A	
A 90% confidence interval		
$\approx\left(51.5625-1.645\times\frac{9}{\sqrt{16}}, 51.5625+1.645\times\frac{9}{\sqrt{16}}\right)$	1M+1A	
$=(47.86125, 55.26375)$	1A	OR $(47.8613, 55.2638)$
(ii) Let n be the sample size.		
$\therefore\ 2\left(1.645\cdot\frac{9}{\sqrt{n}}\right)<6$	1M	
$n>24.354225$	1A	
Hence, the least sample size is 25 .	1A	
	(7)	
(b) (i) P(a tourist waits for more than 65 minutes)		
$=\mathrm{P}\left(Z>\frac{65-51.5}{9}\right)$	1M	
$=\mathrm{P}(Z>1.5)$		
≈ 0.0668	1A	
P(less than 2 coupons are sent to the first 10 tourists interviewed)		
$\approx(1-0.0668)^{10}+C_1^{10}(1-0.0668)^9(0.0668)$	1M	
≈ 0.8594	1A	
(ii) P(the 5th coupon is sent to the 20th tourist interviewed)		
$\approx C_4^{19}(1-0.0668)^{15}(0.0668)^4\cdot 0.0668$	1M	
≈ 0.0018	1A	
	(6)	

Solution	Marks	Remarks
13. (a) P(at least 2 drunk drivers are prosecuted)		
$=1-e^{-2.3}-e^{-2.3}(2.3)$	1A	
≈ 0.669145815		
≈ 0.6691	1A	
	(2)	
(b) $P(\leq 4 \text{ drunk drivers are prosecuted} \mid \text{at least } 2 \text{ drunk drivers are prosecuted})$		
$\approx \dfrac{e^{-2.3}\left(\dfrac{2.3^2}{2!}+\dfrac{2.3^3}{3!}+\dfrac{2.3^4}{4!}\right)}{0.669145815}$	1M+1M	1M for Poisson 1M for conditional prob
≈ 0.8748	1A	
	(3)	
(c) (i) P(the third night was the 1st night to have ≥ 2 drunk drivers prosecuted)		
$\approx (1-0.669145815)^2(0.669145815)$	1M	
≈ 0.0732	1A	
(ii) P(≥ 2 drunk drivers prosecuted in each night and totally 10 prosecuted)		
$=C_2^3\left(e^{-2.3}\dfrac{2.3^2}{2!}\right)^2\left(e^{-2.3}\dfrac{2.3^6}{6!}\right)+3!\left(e^{-2.3}\dfrac{2.3^2}{2!}\right)\left(e^{-2.3}\dfrac{2.3^3}{3!}\right)\left(e^{-2.3}\dfrac{2.3^5}{5!}\right)$ $+C_2^3\left(e^{-2.3}\dfrac{2.3^2}{2!}\right)\left(e^{-2.3}\dfrac{2.3^4}{4!}\right)^2+C_2^3\left(e^{-2.3}\dfrac{2.3^3}{3!}\right)^2\left(e^{-2.3}\dfrac{2.3^4}{4!}\right)$	1M+1M	1M for any one case 1M for all cases
≈ 0.0471	1A	
	(5)	