2012-DSE
MATH EP
M1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2012

MATHEMATICS Extended Part
Module 1 (Calculus and Statistics)

Question-Answer Book

8.30 am – 11.00 am (2½ hours)
This paper must be answered in English

Please stick the barcode label here.

Candidate Number									

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5 and 7.

2. This paper consists of Section A and Section B. Answer **ALL** questions in this paper.

3. Write your answers for Section A in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Write your answers for Section B in the DSE(B) answer book. Start each question (not part of a question) on a new page.

5. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** the book.

6. The Question-Answer book and the answer book will be collected separately at the end of the examination.

7. Unless otherwise specified, all working must be clearly shown.

8. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

9. The diagrams in this paper are not necessarily drawn to scale.

10. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

Section A (50 marks)
In this section, write your answers in the spaces provided in this Question-Answer Book.

1. Let n be a positive integer.

   (a) Expand $(1+3x)^n$ in ascending powers of x up to the term x^2.

   (b) It is given that the coefficient of x^2 in the expansion of $e^{-2x}(1+3x)^n$ is 62. Find the value of n.

(4 marks)

2. The rate of change of the value V (in million dollars) of a flat is given by $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{t}{\sqrt{4t+1}}$, where t is the number of years since the beginning of 2012. The value of the flat is 3 million dollars at the beginning of 2012. Find the percentage change in the value of the flat from the beginning of 2012 to the beginning of 2014.

(5 marks)

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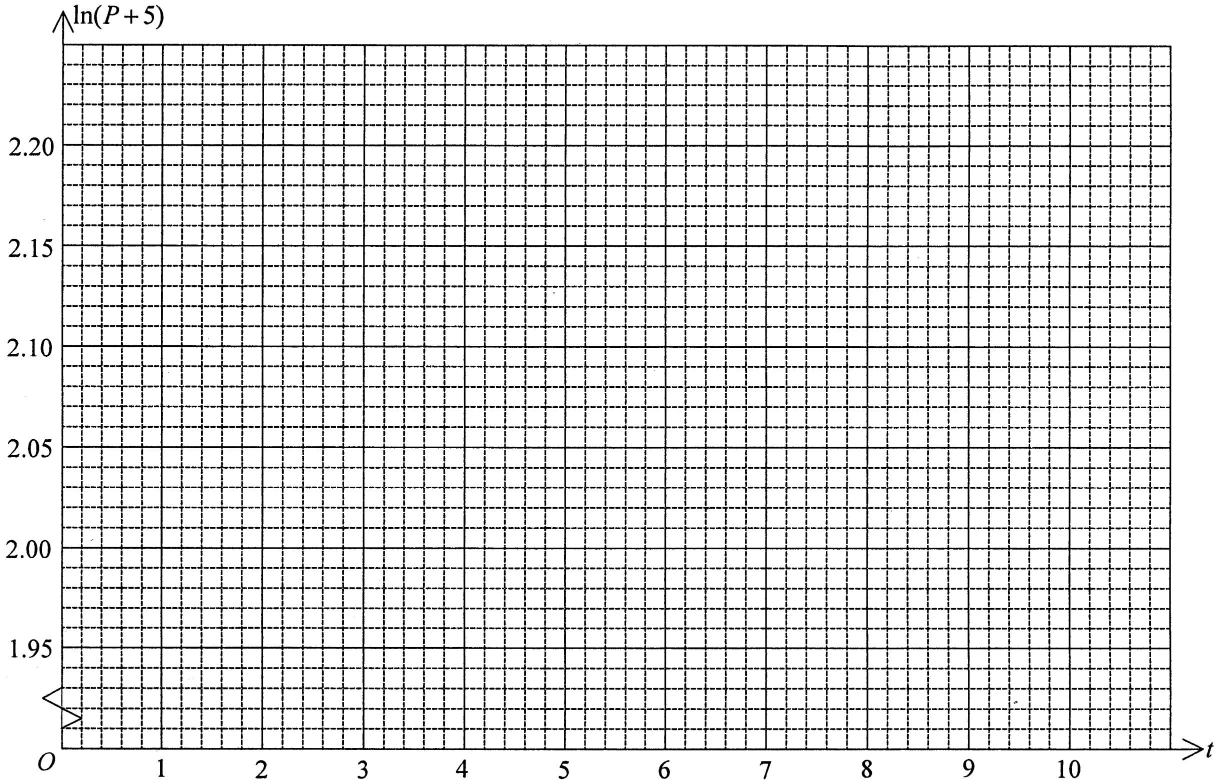
3. The population P (in millions) of a city can be modelled by $P = ae^{\frac{kt}{40}} - 5$, where a and k are constants and t is the number of years since the beginning of a certain year. The population of the city is recorded as follows.

t	2	4	6	8	10
P	2.36	2.81	3.23	3.55	4.01

(a) Express $\ln(P+5)$ as a linear function of t.

(b) Using the graph paper below, estimate the values of a and k. Correct your answers to the nearest integers.

(5 marks)



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4. Let $y=\sqrt[3]{\dfrac{3x-1}{x-2}}$, where $x>2$.

(a) Use logarithmic differentiation to express $\dfrac{1}{y}\cdot\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x .

(b) Using the result of (a), find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ when $x=3$.

(6 marks)

5. The slope of the tangent to a curve S at any point (x, y) on S is given by $\dfrac{\mathrm{d}y}{\mathrm{d}x}=e^{2x}$. Let L be the tangent to S at the point $A(0, 1)$ on S .

(a) Find the equation of S .

(b) Find the equation of L .

(c) Find the area of the region bounded by S , L and the line $x=1$.

(7 marks)

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6. The weights (in kg) of the students in a school can be modelled by the normal distribution with mean 67 and standard deviation 15. A random sample of 36 students is taken.

(a) Find the probability that the mean weight of the 36 students is over 70 kg.

(b) It is found that 9 students in the sample like French fries. Find an approximate 95% confidence interval for the proportion of students in the school who like French fries.

(5 marks)

7. The number of goals scored in a randomly selected match by a football team follows a Poisson distribution with mean λ. The probability that the team scores no goals in a match is 0.1653.

(a) Find the value of λ correct to 1 decimal place.

(b) Find the probability that the team scores less than 3 goals in a match.

(c) It is known that the numbers of goals scored by the team in any two matches are independent. Find the probability that the team totally scores less than 3 goals in two randomly selected matches.

(5 marks)

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8. Let X be a discrete random variable with probability function shown below:

x	1	3	4	6	9	13
$P(X=x)$	0.1	a	0.25	0.15	b	0.05

where a and b are constants. It is known that $E(X)=5.5$.

(a) Find the values of a and b .

(b) Let F be the event that $X \geq 4$ and G be the event that $X<8$.

(i) Find $P(F \cap G)$.

(ii) Are F and G independent events? Justify your answer.

(6 marks)

9. Among the students sitting for a Mathematics test, 73% of them had revised before the test. For those who had revised, their scores are real numbers which can be modelled by $N(59,10^2)$; and for those who had not revised, their scores are real numbers which can be modelled by $N(35.2,12^2)$. Students who scored at least 43 passed the test.

(a) Find the probability that a randomly selected student passed the test.

(b) Given that a randomly selected student passed the test, find the probability that he had not revised before the test.

(c) Ten students are randomly selected among those who passed the test. Find the probability that exactly four of them had not revised before the test.

(7 marks)

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Section B (50 marks)
In this section, write your answers in the DSE(B) answer book.

10. Let $I=\int_1^4 \frac{1}{\sqrt{t}} e^{\frac{-t}{2}}\,\mathrm{d}t$.

(a) (i) Use the trapezoidal rule with 6 sub-intervals to estimate I .

(ii) Is the estimate in (a)(i) an over-estimate or under-estimate? Justify your answer.

(7 marks)

(b) Using a suitable substitution, show that $I=2\int_1^2 e^{\frac{-x^2}{2}}\,\mathrm{d}x$.

(3 marks)

(c) Using the above results and the Standard Normal Distribution Table on page 14, show that $\pi<3.25$.

(3 marks)

11. In a research of the radiation intensity of a city, an expert modelled the rate of change of the radiation intensity R (in suitable units) by

$$\frac{\mathrm{d}R}{\mathrm{d}t}=\frac{a(30-t)+10}{(t-35)^2+b}$$

where t $(0\le t\le T)$ is the number of days elapsed since the start of the research, a , b and T are positive constants.

It is known that the intensity increased to the greatest value of 6 units at $t=35$, and then decreased to the level as at the start of the research at $t=T$. Moreover, the decrease of the intensity from $t=40$ to $t=41$ is $\ln\frac{61}{50}$ units.

(a) Find the value of a .

(2 marks)

(b) Find the value of T .

(4 marks)

(c) Express R in terms of t .

(4 marks)

(d) For $0\le t\le 35$, when would the rate of change of the radiation intensity attain its greatest value?

(4 marks)

12. A company provides cable-car service for tourists. Tourists complain that the waiting time for the cable-car is too long. From past experience, the waiting time (in minutes) of a randomly selected tourist follows a normal distribution with mean μ and standard deviation 9 .

(a) The customer service manager of the company conducts a survey on the waiting time to estimate μ .

(i) A random sample of 16 tourists is taken and their waiting times are recorded as below:

56	36	48	63	57	41	50	43
56	55	62	46	55	69	38	50

Construct a 90% confidence interval for μ .

(ii) Find the least sample size to be taken such that the width of the 90% confidence interval for μ is less than 6 minutes.

(7 marks)

(b) Suppose that $\mu = 51.5$. The customer service manager of the company interviews tourists and will give a coupon to a tourist whose waiting time is more than 65 minutes.

(i) Find the probability that he gives less than 2 coupons to the first 10 tourists interviewed.

(ii) Find the probability that the 5th coupon is given to the 20th tourist interviewed.

(6 marks)

13. Drunk driving is against the law in a city. The police set up an inspection block at the entrance of a certain highway at night in order to arrest drunk drivers. From past experience, the number of drunk drivers arrested follows a Poisson distribution with mean 2.3 per hour.

(a) Find the probability that at least 2 drunk drivers are arrested in a certain hour.

(2 marks)

(b) Given that at least 2 drunk drivers are arrested in a certain hour, find the probability that not more than 4 drunk drivers are arrested.

(3 marks)

(c) In a certain week, the police sets up an inspection block for three nights, all at the same period from 1:00 am to 2:00 am. It is known that the numbers of drunk drivers arrested in different nights are independent.

(i) Find the probability that the third night is the first night to have at least 2 drunk drivers arrested.

(ii) Find the probability that at least 2 drunk drivers are arrested in each of the 3 nights and there are totally 10 drunk drivers arrested.

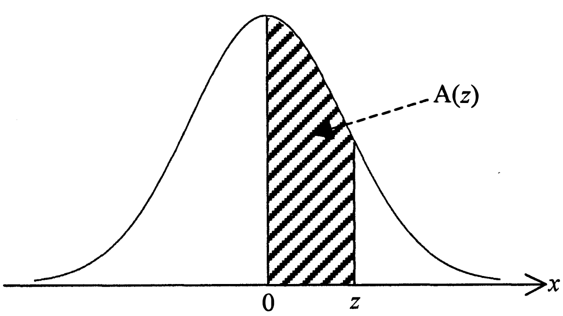
(5 marks)

END OF PAPER

Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ $(z \geq 0)$. Areas for negative values of z can be obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$