HKDSE Maths (M1) 2017

Solution	Marks	Remarks
1. (a) $k^2+0.16+0.18+0.3+k+0.12=1$	1M	
$k^2+k-0.24=0$		
$k=0.2$ or $k=-1.2$ (rejected)	1A	
Thus, we have $k=0.2$.		
(b) $\mathrm{E}(X)$		
$=0(0.04)+2(0.16)+4(0.18)+5(0.3)+8(0.2)+9(0.12)$	1M	
$=5.22$	1A	
(c) $\mathrm{Var}(2-3X)$		
$=9\mathrm{Var}(X)$		
$=9((0-5.22)^2(0.04)+(2-5.22)^2(0.16)+(4-5.22)^2(0.18)$		
$+(5-5.22)^2(0.3)+(8-5.22)^2(0.2)+(9-5.22)^2(0.12))$	1M	
$=56.6244$	1A	
$\mathrm{Var}(2-3X)$		
$=9\mathrm{Var}(X)$		
$=9(\mathrm{E}(X^2)-(\mathrm{E}(X))^2)$		
$=9(33.54-(5.22)^2)$	1M	
$=56.6244$	1A	
	----------(6)	
2. (a) $\mathrm{P}(B\mid A)$		
$=\dfrac{\mathrm{P}(A\mid B)\mathrm{P}(B)}{\mathrm{P}(A)}$		
$=\dfrac{\mathrm{P}(A\mid B)(1-\mathrm{P}(B'))}{\mathrm{P}(A)}$		
$=\dfrac{0.6(1-0.7)}{0.2}$	1M	
$=0.9$	1A	
(b) $\mathrm{P}(A\cap B)$		
$=\mathrm{P}(A\mid B)\mathrm{P}(B)$		
$=\mathrm{P}(A\mid B)(1-\mathrm{P}(B'))$		
$=0.6(1-0.7)$		
$=0.18$		
$\neq 0$	1M	
Thus, A and B are not mutually exclusive.	1A	f.t.
(c) Note that $\mathrm{P}(A\mid B)=0.6\neq 0.2=\mathrm{P}(A)$.	1M	
Thus, A and B are not independent.	1A	f.t.
Note that $\mathrm{P}(A\cap B)=0.18\neq 0.06=\mathrm{P}(A)\,\mathrm{P}(B)$.	1M	
Thus, A and B are not independent.	1A	f.t.
	----------(6)	

Solution	Marks	Remarks
3. (a) μ		
$=\dfrac{1.83+3.43}{2}$		
$=2.63$	1A	
$\mathrm{P}\left(\dfrac{1.83-2.63}{\sigma}<Z<\dfrac{3.43-2.63}{\sigma}\right)=0.8904$	1M	
$\mathrm{P}\left(\dfrac{-0.8}{\sigma}<Z<\dfrac{0.8}{\sigma}\right)=0.8904$		
$\mathrm{P}\left(0<Z<\dfrac{0.8}{\sigma}\right)=0.4452$		
$\dfrac{0.8}{\sigma}=1.6$		
$\sigma=0.5$	1A	
(b) The required probability		
$=\mathrm{P}\left(\dfrac{2.5-2.63}{\frac{0.5}{\sqrt{9}}}<Z<\dfrac{3.1-2.63}{\frac{0.5}{\sqrt{9}}}\right)$	1M	
$=\mathrm{P}(-0.78<Z<2.82)$		
$=0.2823+0.4976$		
$=0.7799$	1A	
	----------(5)	
4. (a) The required probability		
$=(1-0.6)^3(0.6)$	1M	for $(1-p)^3 p$, $0<p<1$
$=0.0384$	1A	
(b) $1-(1-0.6)^{10-k}>0.95$	1M	for $1-(1-q)^{10-k}$, $0<q<1$
$0.4^{10-k}<0.05$		
$\log(0.4^{10-k})<\log 0.05$	1M	
$k<6.730587608$		
Thus, the greatest value of k is 6 .	1A	
(c) The expected amount of money		
$=15\left(\dfrac{1}{0.6}\right)$	1M	for $15\left(\dfrac{1}{r}\right)$, $0<r<1$
$=\$25$	1A	
	----------(7)	

Solution	Marks	Remarks
5. (a) $(1+e^{3x})^2$		
$=1+2e^{3x}+e^{6x}$	1M	
$=1+2\left(1+3x+\frac{(3x)^2}{2!}+\cdots\right)+\left(1+6x+\frac{(6x)^2}{2!}+\cdots\right)$	1M	for expanding e^{3x} or e^{6x}
$=4+12x+27x^2+\cdots$	1A	
$(1+e^{3x})^2$		
$=\left(1+1+3x+\frac{(3x)^2}{2!}+\cdots\right)^2$	1M	for expanding e^{3x}
$=(2)(2)+(2)(2)(3x)+(3x)(3x)+(2)(2)\left(\frac{9x^2}{2}\right)+\cdots$	1M	
$=4+12x+27x^2+\cdots$	1A	
(b) $(5-x)^4$		
$=5^4-C_1^4(5^3)x+C_2^4(5^2)x^2-C_3^4(5)x^3+x^4$	1M	
$=625-500x+150x^2-20x^3+x^4$		
The required coefficient		
$=(625)(27)+(-500)(12)+(150)(4)$	1M	withhold 1M if the step is skipped
$=11\,475$	1A	
	----------(6)	
6. (a) $\mathrm{f}(6)=-33$		
$4(6^3)+m(6^2)+n(6)+615=-33$	1M	
$6m+n=-252$		
$\mathrm{f}'(x)=12x^2+2mx+n$		
$\mathrm{f}'(6)=0$		
$12(6^2)+2m(6)+n=0$	1M	
$12m+n=-432$		
Solving, we have $m=-30$ and $n=-72$.	1A	for both correct
(b) $\mathrm{f}'(x)=12x^2-60x-72$		
$\mathrm{f}'(x)=0$ when $x=-1$ or $x=6$.	1M	
(table below)	1M	for testing
Thus, the minimum value is -33 and the maximum value is 653 .	1A	for both correct
	----------(6)	

x	$(-\infty,-1)$	-1	$(-1,6)$	6	$(6,\infty)$
$\mathrm{f}'(x)$	+	0	−	0	+
$\mathrm{f}(x)$	↗	653	↘	−33	↗

Solution	Marks	Remarks
7. (a) $y=\dfrac{x}{\sqrt{x-2}}$		
$\dfrac{dy}{dx}=\dfrac{\sqrt{x-2}-x\left(\frac{1}{2}\right)(x-2)^{\frac{-1}{2}}}{x-2}$	1M	for quotient rule
$\dfrac{dy}{dx}=\dfrac{x-4}{2(x-2)^{\frac{3}{2}}}$	1A	
(b) Let (h,k) be the coordinates of the point of contact.		
So, the slope of this tangent is $\dfrac{h-4}{2(h-2)^{\frac{3}{2}}}$.		
$\dfrac{k-0}{h-9}=\dfrac{h-4}{2(h-2)^{\frac{3}{2}}}$	1M+1A	1M for using (a)
$\dfrac{h}{\sqrt{h-2}}2(h-2)^{\frac{3}{2}}=(h-4)(h-9)$		
$h^2+9h-36=0$	1M	
$h=3$ or $h=-12$ (rejected)		
The slope of this tangent		
$=\dfrac{3-4}{2(3-2)^{\frac{3}{2}}}$	1M	
$=\dfrac{-1}{2}$	1A	
	----------(7)	
8. (a) Let $u=\ln x$.	1M	
So, we have $\dfrac{du}{dx}=\dfrac{1}{x}$.		
$\int g(x)dx$		
$=\int\left(\dfrac{1}{x}\ln\left(\dfrac{e}{x}\right)\right)dx$		
$=\int\left(\dfrac{1}{x}(1-\ln x)\right)dx$		
$=\int(1-u)du$	1M	
$=u-\dfrac{1}{2}u^2+\text{constant}$		
$=\ln x-\dfrac{1}{2}(\ln x)^2+\text{constant}$	1A	

Solution	Marks	Remarks
Let $u=\ln\left(\frac{e}{x}\right)$.	1M	
Then, we have $\frac{du}{dx}=\frac{-1}{x}$.		
$\int g(x)\,dx$		
$=\int\left(\frac{1}{x}\ln\left(\frac{e}{x}\right)\right)dx$		
$=\int -u\,du$	1M	
$=\frac{-1}{2}u^2+\text{constant}$		
$=\frac{-1}{2}\left(\ln\left(\frac{e}{x}\right)\right)^2+\text{constant}$	1A	
(b) (i) e	1A	
(ii) The required area		
$=\int_1^e g(x)\,dx+\int_e^{e^2}-g(x)\,dx$	1M	
$=\left[\ln x-\frac{1}{2}(\ln x)^2\right]_1^e+\left[-\ln x+\frac{1}{2}(\ln x)^2\right]_e^{e^2}$ (by (a))	1M	for using (a)
$=\frac{1}{2}+\frac{1}{2}$		
$=1$	1A	
The required area		
$=\int_1^e g(x)\,dx+\int_e^{e^2}-g(x)\,dx$	1M	
$=\left[\frac{-1}{2}\left(\ln\left(\frac{e}{x}\right)\right)^2\right]_1^e+\left[\frac{1}{2}\left(\ln\left(\frac{e}{x}\right)\right)^2\right]_e^{e^2}$ (by (a))	1M	for using (a)
$=\frac{1}{2}+\frac{1}{2}$		
$=1$	1A	
	----------(7)	

Solution	Marks	Remarks
9. (a) (i) The sample mean		
$= \dfrac{(0.75)(11)+(1.25)(13)+(1.75)(8)+(2.25)(5)+(2.75)(3)}{40}$		
$= 1.45$ hours	1A	
A 90% confidence interval for μ		
$= \left(1.45 - 1.645\left(\dfrac{0.4}{\sqrt{40}}\right), 1.45 + 1.645\left(\dfrac{0.4}{\sqrt{40}}\right)\right)$	1M+1A	1A for 1.645
$\approx (1.3460, 1.5540)$	1A	r.t. (1.3460, 1.5540)
(ii) Let n be the sample size.		
$2(2.17)\left(\dfrac{0.4}{\sqrt{n}}\right) \le 0.3$	1M+1A	1A for 2.17
$n \ge 33.48551111$		
Thus, the least sample size is 34 .	1A ----------(7)	
(b) (i) The required probability		
$= \text{P}\left(Z > \dfrac{2-1.48}{0.4}\right)$	1M	
$= \text{P}(Z > 1.3)$		
$= 0.5 - 0.4032$		
$= 0.0968$	1A	
(ii) The required probability		
$= \dfrac{C_1^9(1-0.0968)^8(0.0968)^2}{1-(1-0.0968)^{15} - C_1^{15}(1-0.0968)^{14}(0.0968)}$	1M+1M+1M	1M for using (b)(i) + 1M for numerator + 1M for denominator
≈ 0.086102962		
≈ 0.0861	1A ----------(6)	r.t. 0.0861

Solution	Marks	Remarks
10. (a) The required probability		
$=\frac{2^0e^{-2}}{0!}+\frac{2^1e^{-2}}{1!}+\frac{2^2e^{-2}}{2!}+\frac{2^3e^{-2}}{3!}+\frac{2^4e^{-2}}{4!}$	1M+1M	1M for the 5 cases + 1M for Poisson probability
≈ 0.947346982		
≈ 0.9473	1A ----------(3)	r.t. 0.9473
(b) The required probability		
$=\frac{2^3e^{-2}}{3!}(3(0.25)^2(0.1)+3(0.25)(0.2)^2+3(0.45)^2(0.2))$	1M+1M	1M for Poisson probability + 1M for any one correct
≈ 0.030721109		
≈ 0.0307	1A ----------(3)	r.t. 0.0307
(c) The required probability		
$=4(0.25)^3(0.1)+6(0.25)^2(0.2)^2+(4)(3)(0.45)^2(0.2)(0.25)+(0.45)^4$	1M+1M	1M for any one correct + 1M for any three correct
$=0.18375625$		
≈ 0.1838	1A ----------(3)	r.t. 0.1838
(d) The required probability		
$\approx\frac{\left(\frac{2e^{-2}}{1!}\right)(0.1)+\left(\frac{2^2e^{-2}}{2!}\right)(2(0.25)(0.1)+(0.2)^2)+0.030721109+\left(\frac{2^4e^{-2}}{4!}\right)(0.18375625)}{0.947346982}$	1M+1M	1M for numerator using (b) or (c) +1M for denominator using (a)
≈ 0.10421488		
≈ 0.1042	1A ----------(3)	r.t. 0.1042

Solution	Marks	Remarks
11. (a) According to the suggestion by Ada,		
I		
$\approx \frac{1}{2}\left(\frac{1-0.5}{5}\right)(\mathrm{f}(0.5)+\mathrm{f}(1)+2(\mathrm{f}(0.6)+\mathrm{f}(0.7)+\mathrm{f}(0.8)+\mathrm{f}(0.9)))$	1M	
≈ 0.747559672		
≈ 0.7476	1A	r.t. 0.7476
According to the suggestion by Billy,		
I		
$\approx \int_{0.5}^{1}\left(\frac{1}{x}+0.1+0.005x\right)dx$	1M	
$=\left[\ln x+0.1x+0.0025x^2\right]_{0.5}^{1}$	1M	
$=\ln 2+0.051875$	1A	r.t. 0.7450
≈ 0.74502218		
≈ 0.7450		
	----------(5)	
(b) $\mathrm{f}(x)$		
$=\frac{e^{0.1x}}{x}$		
$\mathrm{f}'(x)$		
$=\frac{0.1\,e^{0.1x}}{x^2}(x-10)$	1M	
$\mathrm{f}''(x)$		
$=\frac{0.01\,e^{0.1x}}{x^3}(x^2-20x+200)$	1A	
$=\frac{0.01\,e^{0.1x}}{x^3}((x-10)^2+100)$	1M	
>0 for $0.5\le x\le 1$.		
Thus, the estimate suggested by Ada is an over-estimate.	1A	f.t.
$e^{0.1x}=1+0.1x+\frac{(0.1x)^2}{2!}+\frac{(0.1x)^3}{3!}+\cdots$	1M	
$e^{0.1x}>1+0.1x+0.005x^2$ for $0.5\le x\le 1$		
$I>\int_{0.5}^{1}\left(\frac{1}{x}+0.1+0.005x\right)dx$		
Thus, the estimate suggested by Billy is an under-estimate.	1A	f.t.
	----------(6)	
(c) $0.7450<I<0.7476$	1M	
$-0.0010<I-0.746<0.0016$		
So, we have $-0.002<I-0.746<0.002$.		
Thus, the claim is agreed.	1A	f.t.
$I-0.746<0.7476-0.746=0.0016$	1M	
$0.746-I<0.746-0.7450=0.0010$		
So, the difference of I and 0.746 is less than 0.002 .		
Thus, the claim is agreed.	1A	f.t.
	----------(2)	

Solution	Marks	Remarks
12. (a) (i) $x-4=\dfrac{3k}{2^{\lambda t}-k}$		
$x-1=\dfrac{3(2^{\lambda t})}{2^{\lambda t}-k}$		
$(x-4)(x-1)=\dfrac{9k2^{\lambda t}}{(2^{\lambda t}-k)^2}$	1A	
(ii) $\dfrac{9k2^{\lambda t}}{(2^{\lambda t}-k)^2}>0$ (as $k>0$)		
$(x-4)(x-1)>0$ (by (a)(i))	1M	
$x>4$ or $x<1$		
Thus, the claim is correct.	1A	f.t.
	----------(3)	
(b) (i) $\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{-3(\ln 2)k\lambda 2^{\lambda t}}{(2^{\lambda t}-k)^2}$	1A	
$\dfrac{-\ln 2}{24}(x-4)(x-1)=\dfrac{-3(\ln 2)k2^{\lambda t}}{8(2^{\lambda t}-k)^2}$		
$\lambda=\dfrac{1}{8}$	1	
(ii) (1) When $t=0$, $x=0.8$.		
$-3.2=\dfrac{3k}{1-k}$		
$k=16$	1A	
When $x=0$, we have $4+\dfrac{48}{2^{\frac{t}{8}}-16}=0$.	1M	either one
So, we have $2^{\frac{t}{8}}=4$.	1M	either one
Solving, we have $t=16$.		
Thus, the crocodiles in the lake will eventually become extinct in 16 years.	1A	
(2) When $t=0$, $x=7$.		
$3=\dfrac{3k}{1-k}$		
$k=0.5$	1A	
When $x=0$, we have $4+\dfrac{1.5}{2^{\frac{t}{8}}-0.5}=0$.		
So, we have $2^{\frac{t}{8}}=0.125$.		
It is impossible as $2^{\frac{t}{8}}>1$ for $t>0$.		
Thus, the crocodiles in the lake will never become extinct.	1A	f.t.
Note that $\lim\limits_{t\to\infty}x=\lim\limits_{t\to\infty}\left(4+\dfrac{1.5}{2^{\frac{t}{8}}-0.5}\right)=4$.		
After a very long time, the estimated number of crocodiles in the lake is 4 000 .	1A	
	----------(9)	