Learning Content of Module 1 (Calculus and Statistics)

Notes:

1. *Learning units are grouped under three areas ("Foundation Knowledge", "Calculus" and "Statistics") and a Further Learning Unit.*
2. *Related learning objectives are grouped under the same learning unit.*
3. *The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.*
4. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*

Learning Unit	Learning Objective	Time	Remarks
Foundation Knowledge			
1. Binomial expansion	1.1 recognise the expansion of $(a+b)^n$, where n is a positive integer	3	Students are required to recognise the summation notation ($\sum$). The following contents are **not** required: • expansion of trinomials • the greatest coefficient, the greatest term and the properties of binomial coefficients • applications to numerical approximation

Learning Unit	Learning Objective	Time	Remarks
2. Exponential and logarithmic functions	2.1 recognise the definition of e and the exponential series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$	8	
	2.2 understand exponential functions and logarithmic functions		The following functions are required: • $y = e^x$ • $y = \ln x$
	2.3 use exponential functions and logarithmic functions to solve problems		Students are required to solve problems including those related to compound interest, population growth and radioactive decay.
	2.4 transform $y = ka^x$ and $y = k[f(x)]^n$ to linear relations, where a , n and k are real numbers, $a > 0$, $a \neq 1$, $f(x) > 0$ and $f(x) \neq 1$		When experimental values of x and y are given, students are required to plot the graph of the corresponding linear relation from which they can determine the values of the unknown constants by considering its slope and intercepts.
	Subtotal in hours	11	

Learning Unit	Learning Objective	Time	Remarks
Calculus			
3. Derivative of a function	3.1 recognise the intuitive concept of the limit of a function	5	Student are required to recognise the theorems on the limits of sum, difference, product, quotient, scalar multiplication of functions and the limits of composite functions (the proofs are **not** required).
	3.2 find the limits of algebraic functions, exponential functions and logarithmic functions		The following algebraic functions are required: ● polynomial functions ● rational functions ● power functions x^α ● functions derived from the above ones through addition, subtraction, multiplication, division and composition, such as $\sqrt{x^2+1}$
	3.3 recognise the concept of the derivative of a function from first principles		Students are **not** required to find the derivatives of functions from first principles. Students are required to recognise the notations: y', $f'(x)$ and $\frac{dy}{dx}$.

Learning Unit	Learning Objective	Time	Remarks
	3.4 recognise the slope of the tangent of the curve $y = f(x)$ at a point $x = x_0$		Students are required to recognise the notations: $f'(x_0)$ and $\left.\dfrac{dy}{dx}\right\vert_{x=x_0}$.
4. Differentiation of a function	4.1 understand the addition rule, product rule, quotient rule and chain rule of differentiation	8	The rules include: • $\dfrac{d}{dx}(u+v)=\dfrac{du}{dx}+\dfrac{dv}{dx}$ • $\dfrac{d}{dx}(uv)=v\dfrac{du}{dx}+u\dfrac{dv}{dx}$ • $\dfrac{d}{dx}(\dfrac{u}{v})=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$ • $\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$

Learning Unit	**Learning Objective**	**Time**	**Remarks**
	4.2 find the derivatives of algebraic functions, exponential functions and logarithmic functions		The formulae that students are required to use include: • $(C)' = 0$ • $(x^n)' = nx^{n-1}$ • $(e^x)' = e^x$ • $(\ln x)' = \frac{1}{x}$ • $(\log_a x)' = \frac{1}{x \ln a}$ • $(a^x)' = a^x \ln a$ Implicit differentiation and logarithmic differentiation are **not** required.
5. Second derivative	5.1 recognise the concept of the second derivative of a function	2	Students are required to recognise the notations: y'', $f''(x)$ and $\frac{d^2y}{dx^2}$. Third and higher order derivatives are **not** required.

Learning Unit	Learning Objective	Time	Remarks
	5.2 find the second derivative of an explicit function		Students are required to recognise the second derivative test and concavity.
6. Applications of differentiation	6.1 use differentiation to solve problems involving tangent, rate of change, maximum and minimum	10	Local and global extrema are required.
7. Indefinite integration and its applications	7.1 recognise the concept of indefinite integration 7.2 understand the basic properties of indefinite integrals and basic integration formulae	10	Indefinite integration as the reverse process of differentiation should be introduced. Students are required to recognise the notation: $\int f(x)\,dx$. The properties include: • $\int kf(x)dx = k\int f(x)dx$ • $\int [f(x) \pm g(x)]dx$ $= \int f(x)dx \pm \int g(x)dx$ The formulae include: • $\int kdx = kx + C$ • $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

Learning Unit	Learning Objective	Time	Remarks
	7.3 use basic integration formulae to find the indefinite integrals of algebraic functions and exponential functions 7.4 use integration by substitution to find indefinite integrals 7.5 use indefinite integration to solve problems		• $\int \frac{1}{x} dx = \ln\|x\| + C$ • $\int e^x dx = e^x + C$ Students are required to understand the meaning of the constant of integration *C*. Integration by parts is **not** required.
8. Definite integration and its applications	8.1 recognise the concept of definite integration	12	The definition of the definite integral as the limit of a sum of the areas of rectangles under a curve should be introduced. Students are required to recognise the notation: $\int_a^b f(x)\,dx$. The concept of dummy variables is required, for example: $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$.

Learning Unit	Learning Objective	Time	Remarks
	8.2 recognise the Fundamental Theorem of Calculus and understand the properties of definite integrals		The Fundamental Theorem of Calculus that students are required to recognise is: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $\frac{d}{dx}F(x) = f(x)$. The properties include: • $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ • $\int_a^a f(x)\,dx = 0$ • $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ • $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ • $\int_a^b [f(x) \pm g(x)]\,dx$ $= \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
	8.3 find the definite integrals of algebraic functions and exponential functions		
	8.4 use integration by substitution to find definite integrals		

Learning Unit	Learning Objective	Time	Remarks
	8.5 use definite integration to find the areas of plane figures 8.6 use definite integration to solve problems		Students are **not** required to use definite integration to find the area between a curve and the *y*-axis and the area between two curves.
9. Approximation of definite integrals using the trapezoidal rule	9.1 understand the trapezoidal rule and use it to estimate the values of definite integrals	4	Error estimation is **not** required. Students are required to determine whether an estimate is an over-estimate or under-estimate by using the second derivative and concavity.
	Subtotal in hours	51	
Statistics			
10. Conditional probability and Bayes' theorem	10.1 understand the concept of conditional probability 10.2 use Bayes' theorem to solve simple problems	6	
11. Discrete random variables	11.1 recognise the concept of discrete random variables	1	

Learning Unit	Learning Objective	Time	Remarks
12. Probability distribution, expectation and variance	12.1 recognise the concept of discrete probability distribution and represent the distribution in the form of tables, graphs and mathematical formulae	7	
	12.2 recognise the concepts of expectation $E[X]$ and variance $\mathrm{Var}(X)$ and use them to solve simple problems		The formulae that students are required to use include: • $E[X]=\sum xP(X=x)$ • $\mathrm{Var}(X)=E\left[(X-\mu)^2\right]$ • $E[g(X)]=\sum g(x)P(X=x)$ • $E[aX+b]=aE[X]+b$ • $\mathrm{Var}(X)=E\left[X^2\right]-(E[X])^2$ • $\mathrm{Var}(aX+b)=a^2\mathrm{Var}(X)$ Notation $E(X)$ can also be used.

Learning Unit	Learning Objective	Time	Remarks
13. The binomial distribution	13.1 recognise the concept and properties of the binomial distribution	5	The Bernoulli distribution should be introduced. The mean and variance of the binomial distribution are required (the proofs are **not** required).
	13.2 calculate probabilities involving the binomial distribution		Use of the binomial distribution table is **not** required.
14. The Poisson distribution	14.1 recognise the concept and properties of the Poisson distribution	5	The mean and variance of Poisson distribution are required (the proofs are **not** required).
	14.2 calculate probabilities involving the Poisson distribution		Use of the Poisson distribution table is **not** required.
15. Applications of the binomial and the Poisson distributions	15.1 use the binomial and the Poisson distributions to solve problems	5	
16. Basic definition and properties of the normal distribution	16.1 recognise the concepts of continuous random variables and continuous probability distributions, with reference to the normal distribution	3	Derivations of the mean and variance of the normal distribution are **not** required. Students are required to recognise that the formulae in Learning Objective 12.2 are also applicable to continuous random variables.

Learning Unit	Learning Objective	Time	Remarks
	16.2 recognise the concept and properties of the normal distribution		The properties include: ● the curve is bell-shaped and symmetrical about the mean ● the mean, mode and median are all equal ● the flatness can be determined by the value of σ ● the area under the curve is 1
17. Standardisation of a normal variable and use of the standard normal table	17.1 standardise a normal variable and use the standard normal table to find probabilities involving the normal distribution	2	
18. Applications of the normal distribution	18.1 find the values of $P(X > x_1)$, $P(X < x_2)$, $P(x_1 < X < x_2)$ and related probabilities, given the values of x_1, x_2, μ and σ, where $X \sim N(\mu, \sigma^2)$ 18.2 find the values of x, given the values of $P(X > x)$, $P(X < x)$, $P(a < X < x)$, $P(x < X < b)$ or a related probability, where $X \sim N(\mu, \sigma^2)$ 18.3 use the normal distribution to solve problems	7	

Learning Unit	Learning Objective	Time	Remarks
19. Sampling distribution and point estimates	19.1 recognise the concepts of sample statistics and population parameters	9	Students are required to recognise: If the population mean is μ and the population size is N, then the population variance is $\sigma^2 = \dfrac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}$.
	19.2 recognise the sampling distribution of the sample mean $\bar{X}$ from a random sample of size n		Students are required to recognise: • If the population mean is μ and the population variance is σ^2, then $E\left[\bar{X}\right]=\mu$ and $\mathrm{Var}(\bar{X})=\dfrac{\sigma^2}{n}$. • If $X \sim N(\mu,\sigma^2)$, then $\bar{X} \sim N(\mu,\dfrac{\sigma^2}{n})$ (the proof is **not** required).
	19.3 use the Central Limit Theorem to treat $\bar{X}$ as being normally distributed when the sample size n is sufficiently large		

Learning Unit	Learning Objective	Time	Remarks
	19.4 recognise the concept of point estimates including the sample mean and sample variance		Students are required to recognise: If the sample mean is $\overline{x}$ and the sample size is n, then the sample variance is $s^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}$. Students are required to recognise the concept of unbiased estimator.
20. Confidence interval for a population mean	20.1 recognise the concept of confidence interval 20.2 find the confidence interval for a population mean	6	 Students are required to recognise: • A $100(1-\alpha)\%$ confidence interval for the mean μ of a normal population with known variance σ^2, based on a random sample of size n, is given by $(\overline{x} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \overline{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}})$. • When the sample size n is sufficiently large, a $100(1-\alpha)\%$ confidence interval for the mean μ of a population with unknown variance is given by $(\overline{x} - z_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}, \overline{x} + z_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}})$, where s is the

Learning Unit	Learning Objective	Time	Remarks
			sample standard deviation.
	Subtotal in hours	56	
Further Learning Unit			
21. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	7	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.
	Subtotal in hours	7	

Grand total: 125 hours