2012-DSE
MATH EP
M2

Please stick the barcode label here.

Candidate Number									

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2012

MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)

Question-Answer Book

8.30 am – 11.00 am (2½ hours)
This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5 and 7.

2. This paper consists of Section A and Section B. Answer **ALL** questions in this paper.

3. Write your answers for Section A in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Write your answers for Section B in the DSE(B) answer book. Start each question (not part of a question) on a new page.

5. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** the book.

6. The Question-Answer book and the answer book will be collected separately at the end of the examination.

7. Unless otherwise specified, all working must be clearly shown.

8. Unless otherwise specified, numerical answers must be exact.

9. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.

10. The diagrams in this paper are not necessarily drawn to scale.

11. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

****

Answers written in the margins will not be marked.

Section A (50 marks)
In this section, write your answers in the spaces provided in this Question-Answer Book.

1. Let $f(x) = e^{2x}$. Find $f'(0)$ from first principles.

(3 marks)

2. It is given that

$$(1+ax)^n = 1 + 6x + 16x^2 + \text{ terms involving higher powers of } x\ ,$$

where n is a positive integer and a is a constant. Find the values of a and n .

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

3. Prove, by mathematical induction, that for all positive integers n ,

$$1\times 2+2\times 5+3\times 8+\cdots+n(3n-1)=n^2(n+1) \ .$$

(5 marks)

4. (a) Find $\int \frac{x+1}{x}\,dx$.

(b) Using the substitution $u=x^2-1$, find $\int \frac{x^3}{x^2-1}\,dx$.

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

5. Find the minimum point(s) and asymptote(s) of the graph of $y = \frac{x^2 + x + 1}{x + 1}$.

(6 marks)

6.

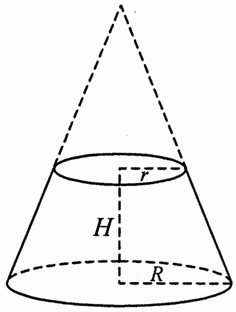

Figure 1

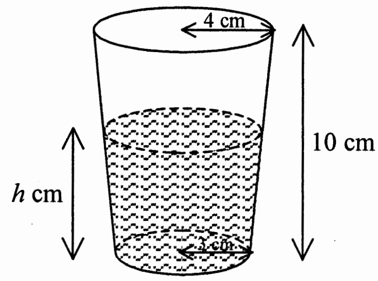

Figure 2

A frustum of height H is made by cutting off a right circular cone of base radius r from a right circular cone of base radius R (see Figure 1). It is given that the volume of the frustum is $\frac{\pi}{3}H(r^2 + rR + R^2)$.

An empty glass is in the form of an inverted frustum described above with height 10 cm , the radii of the rim and the base 4 cm and 3 cm respectively. Water is being poured into the glass. Let h cm $(0 \le h \le 10)$ be the depth of the water inside the glass at time t s (see Figure 2).

(a) Show that the volume V cm^3 of water inside the glass at time t s is given by

$$V = \frac{\pi}{300}(h^3 + 90h^2 + 2700h) .$$

(b) If the volume of water in the glass is increasing at the rate 7π cm^{3}s^{-1} , find the rate of increase of depth of water at the instant when $h = 5$.

(6 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

7.

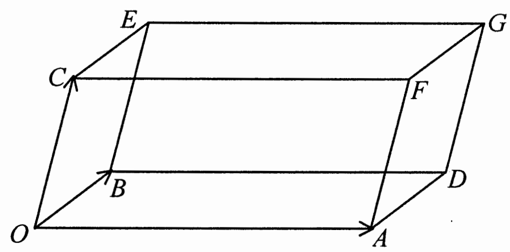

Figure 3

Figure 3 shows a parallelepiped $OADBECFG$. Let $\overrightarrow{OA} = 6\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\overrightarrow{OB} = 2\mathbf{i} + \mathbf{j}$ and $\overrightarrow{OC} = 5\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

(a) Find the area of the parallelogram $OADB$.

(b) Find the distance between point C and the plane $OADB$.

(5 marks)

8. (a) Solve the following system of linear equations:

$$\begin{cases} x + y + z = 0 \\ 2x - y + 5z = 6 \end{cases} .$$

(b) Using (a), or otherwise, solve the following system of linear equations:

$$\begin{cases} x + y + z = 0 \\ 2x - y + 5z = 6 \\ x - y + \lambda z = 4 \end{cases} ,$$ where λ is a constant.

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

9. (a) Using integration by parts, find $\int x \sin x dx$.

(b)

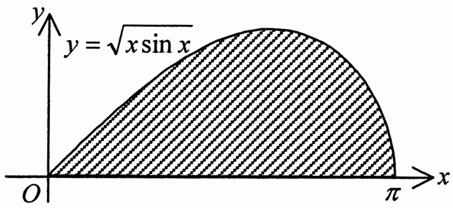

Figure 4

Figure 4 shows the shaded region bounded by the curve $y=\sqrt{x\sin x}$ for $0 \le x \le \pi$ and the x-axis. Find the volume of the solid generated by revolving the region about the x-axis.

(4 marks)

10.

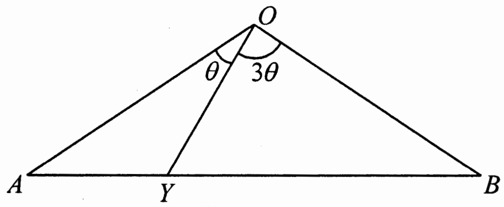

Figure 5

In Figure 5, OAB is an isosceles triangle with $OA = OB$, $AB = 1$, $AY = y$, $\angle AOY = \theta$ and $\angle BOY = 3\theta$.

(a) Show that $y = \frac{1}{4}\sec^2\theta$.

(b) Find the range of values of y .

[Hint: you may use the identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.]

(6 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Section B (50 marks)
In this section, write your answers in the DSE(B) answer book.

11. (a) Solve the equation

$$\begin{vmatrix} 1-x & 4 \\ 2 & 3-x \end{vmatrix} = 0 \text{ ---------------------------------- } (*) .$$

(2 marks)

(b) Let x_1, x_2 $(x_1 < x_2)$ be the roots of (*). Let $P = \begin{pmatrix} a & c \\ b & 1 \end{pmatrix}$. It is given that

$$\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = x_1\begin{pmatrix} a \\ b \end{pmatrix} , \quad \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} c \\ 1 \end{pmatrix} = x_2\begin{pmatrix} c \\ 1 \end{pmatrix} \quad \text{and} \quad |P| = 1 ,$$

where a, b and c are constants.

(i) Find P.

(ii) Evaluate $P^{-1}\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}P$.

(iii) Using (b)(ii), evaluate $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}^{12}$.

(11 marks)

12.

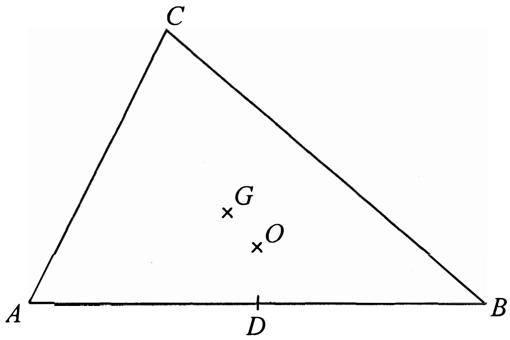

Figure 6

Figure 6 shows an acute angled scalene triangle ABC, where D is the mid-point of AB, G is the centroid and O is the circumcentre. Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$.

(a) Express $\overrightarrow{AG}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

(3 marks)

(b) It is given that E is a point on AB such that CE is an altitude. Extend OG to meet CE at F.

(i) Prove that $\Delta DOG \sim \Delta CFG$.

Hence find $FG : GO$.

(ii) Show that $\overrightarrow{AF} = \mathbf{b} + \mathbf{c}$.

Hence prove that F is the orthocentre of ΔABC .

(9 marks)

13. (a) (i) Suppose $\tan u = \dfrac{-1+\cos\dfrac{2\pi}{5}}{\sin\dfrac{2\pi}{5}}$, where $\dfrac{-\pi}{2} < u < \dfrac{\pi}{2}$.

Show that $u = \dfrac{-\pi}{5}$.

(ii) Suppose $\tan v = \dfrac{1+\cos\dfrac{2\pi}{5}}{\sin\dfrac{2\pi}{5}}$.

Find v, where $\dfrac{-\pi}{2} < v < \dfrac{\pi}{2}$.

(4 marks)

(b) (i) Express $x^2 + 2x\cos\dfrac{2\pi}{5} + 1$ in the form $(x+a)^2 + b^2$, where a and b are constants.

(ii) Evaluate $\displaystyle\int_{-1}^{1} \frac{\sin\dfrac{2\pi}{5}}{x^2 + 2x\cos\dfrac{2\pi}{5} + 1}\,dx$.

(6 marks)

(c) Evaluate $\displaystyle\int_{-1}^{1} \frac{\sin\dfrac{7\pi}{5}}{x^2 + 2x\cos\dfrac{7\pi}{5} + 1}\,dx$.

(3 marks)

14. Consider the curve Γ: $y = kx^p$, where $k > 0$, $p > 0$. In Figure 7, the tangent to Γ at $A(a, ka^p)$ cuts the x-axis at $B(-a, 0)$, where $a > 0$.

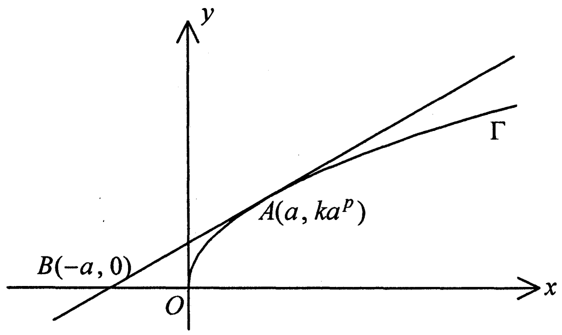

Figure 7

(a) Show that $p = \frac{1}{2}$.

(3 marks)

(b) Suppose that $a = 1$. As shown in Figure 8, the circle C, with radius 2 and centre on the y-axis, touches Γ at point A .

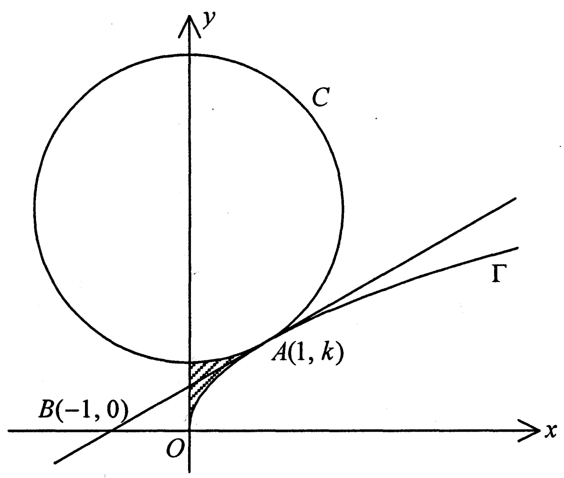

Figure 8

(i) Show that $k = \frac{2\sqrt{3}}{3}$.

(ii) Find the area of the shaded region bounded by Γ , C and the y-axis.

(9 marks)

END OF PAPER