Candidates' performance

Module 2

Section A

Question Number	Performance in General
1.	Satisfactory. Most candidates knew the fundamental formula in finding derivative from first principles and employed a 'sum to product formula' to deal with the trigonometric expression. Many of them, however, lost marks because they did not show explicitly that $\lim_{h\to 0}\frac{\sin h}{h}=1$.
2.	Very Good. However, some candidates lost marks as they did not show the steps in finding n and a . A few candidates did not express $C_2^n=\frac{n(n-1)}{2}$ correctly.
3.	Very Good. Some candidates transformed the proposition to be $\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\cdots+\frac{1}{(3n-2)(3n+1)}=\frac{4n+1}{3n+1}-1$ and got the correct proof. Some common mistakes included the following: ■ For $n=1$, L.H.S. $=\frac{1}{1\times 4}=\frac{1}{4}$ and R.H.S. $=\frac{4(1)+1}{3(1)+1}=\frac{5}{4}$; or L.H.S.= 1 and R.H.S. $=\frac{4(1)+1}{3(1)+1}=\frac{5}{4}$ then claimed that the proposition is not true. ■ In the second step, 'Assume the statement is true for all positive integers'. ■ Skipping the essential step in factorization of the numerator of the expression $\cdots=\frac{12k^2+19k+5}{(3k+1)(3k+4)}=\frac{(3k+1)(4k+5)}{(3k+1)(3k+4)}=\cdots$ ■ Not stating 'the statement is true for $n=1$' and/or 'the statement is also true for $n=k+1$' after finishing the first and/or second steps.
4. (a)	Very Good. However, some candidates missed out the arbitrary constant in the answer for indefinite integral and hence failed to find the value of this arbitrary constant by substituting the point $(1, e)$.
(b)	Satisfactory. Some candidates were not able to find the y-intercept correctly. When finding the slope at that point, many candidates wrongly calculated $e^0-1=-1$. Some once find the curve cuts the y-axis at $(0, 2)$ stated that the equation of the tangent is $y=2$ without justification.
5. (a)	Very Good. However, some candidates did not realise that answers could be obtained directly from the table and spent time to find $f'(x)$ and $f''(x)$. A few candidates wrongly mixed up the terms 'maximum point', 'minimum point' and 'point of inflexion'.
(b)	Good. However, some candidates mixed up 'horizontal' and 'vertical' and wrote '$y=-3$ is a vertical asymptote'. After arriving at the correct expression $f(x)=-3+\frac{12}{x^2+3}$, some candidates wrongly stated that '$x=-\sqrt{3}$ and $x=\sqrt{3}$ are the vertical asymptotes', '$x=\pm\sqrt{3}i$ are the asymptotes', etc.
(c)	Satisfactory. Although the first derivative at each point of inflexion was non-zero as indicated in the given table, a number of candidates wrongly sketched two points of inflexion as stationary points. A few candidates missed out all the labelling in the sketch.

6. (a) Very Good. Most candidates possessed basic skills in obtaining integrand of polynomial and finding area by integration. A common mistake was

$$\text{Area}=\int_0^4\left[\left(\frac{-x^2}{2}+2x+4\right)-4\right]dx+\int_4^5\left[5-\left(\frac{-x^2}{2}+2x+4\right)\right]dx\ .$$

(b) Fair. 40% of the candidates scored zero marks in this part. Quite a number of candidates wrongly employed shell method to find the volume. Many candidates used

$\pi\int_0^4\left(\frac{-x^2}{2}+2x+4-4\right)^2dx+\pi\int_4^5\left(\frac{-x^2}{2}+2x+4-4\right)^2dx$ rather than the simple and direct method $\pi\int_0^5\left(\frac{-x^2}{2}+2x+4-4\right)^2dx$ to find the volume.

7. (a) Excellent. Over 90% of the candidates scored full marks. But among them, quite a number of candidates did not prove by using direct process. For instance, instead of directly transforming $\cos 2x=2\cos^2 x-1$, some wrote $\cos 2x=\cos^2 x-\sin^2 x$ and then employed $\cos^2 x=1-\sin^2 x$ to simplify the denominator.

(b) Fair. Many candidates did not apply the identity in (a) appropriately and made mistakes such as $\frac{\sin 8y}{1+\cos 8y}\cdot\frac{\cos 4y}{1+\cos 4y}\cdot\frac{\cos 2y}{1+\cos 2y}=\tan 4y\cdot\tan 2y\cdot\tan y$ or $\frac{\sin 8y}{1+\cos 8y}=\tan y$. Some candidates did not note that they had to use the result of (a) in this proof and hence did not get full mark.

8. (a) Good. About half of the candidates were able to find the inverse of a matrix correctly. Some candidates used the co-factor matrix instead of the adjoint matrix in finding the inverse of M.

Some missed all the negative signs and used $\begin{pmatrix}\begin{vmatrix}1&1\\0&0\end{vmatrix}&\begin{vmatrix}0&1\\k&0\end{vmatrix}&\begin{vmatrix}0&1\\k&0\end{vmatrix}\\\begin{vmatrix}k&0\\0&0\end{vmatrix}&\begin{vmatrix}1&0\\k&0\end{vmatrix}&\begin{vmatrix}1&k\\k&0\end{vmatrix}\\\begin{vmatrix}k&0\\1&1\end{vmatrix}&\begin{vmatrix}1&0\\0&1\end{vmatrix}&\begin{vmatrix}1&k\\0&1\end{vmatrix}\end{pmatrix}$ as the co-factor matrix.

Some candidates mixed up the notations of matrices and determinants and wrote the answer

$$M^{-1}=\frac{1}{k^2}\begin{vmatrix}0&0&k\\k&0&-1\\-k&k^2&1\end{vmatrix}.$$

(b) Satisfactory. Some candidates used methods that were independent of part (a).
Some common mistakes including:

- overlooking the non-commutative property of matrix multiplication and obtaining $\begin{pmatrix}x\\1\\z\end{pmatrix}=\frac{1}{k^2}\begin{pmatrix}2\\2\\1\end{pmatrix}\begin{pmatrix}0&0&k\\k&0&-1\\-k&k^2&1\end{pmatrix}$.
- mixing up M with M^{-1} and stating $\frac{1}{k^2}\begin{pmatrix}0&0&k\\k&0&-1\\-k&k^2&1\end{pmatrix}\begin{pmatrix}x\\1\\z\end{pmatrix}=\begin{pmatrix}2\\2\\1\end{pmatrix}$.

9. (a) Satisfactory. Most candidates started either by applying Gaussian elimination or by considering the determinant of the coefficient matrix. For those who applied Gaussian elimination, some arrived at $(ab-b)z=a+2$ but were not able to conclude that both $ab-b$ and $a+2$ are zero. For those who considered the determinant of the coefficient matrix, some just set $\Delta=0$ but were not aware that $\Delta_z=0$.

(b) Fair. For candidates obtaining full mark in (a), most of them also scored full marks in this part.

10. (a) Very Good. A few candidates lost marks due to wrong memorisation of section formula.

(b) Poor. Some candidates made mistake in the logical reasoning. They thought that if A, N, P, M were concyclic, AP must be diameter and hence claimed that $\angle ANP=\angle AMP=90°$. Another common error was the wrong direction of vector in the dot product formula such as

$$\cos\angle OAB=\frac{\overrightarrow{OA}\cdot\overrightarrow{AB}}{|\overrightarrow{OA}||\overrightarrow{AB}|}\ .$$

11. (a) Very Good. Over three quarters of the candidates scored full marks.

(b) (i) Good. Some candidates lost marks as they used wrong substitution $u=\tan\theta$.

(ii) Poor. Half of the candidates scored no marks in this part. Many candidates did not change the limits of integral. Some lost mark in the negligence of rationalization when arriving at the given answer. Another common mistake was $\sqrt{x^4+4x^2+3}=\sqrt{(x+2)^2-1}$.

(c) Poor. Some candidates did not obtain the correct expression as $\cos^2\phi=\frac{1}{1+t^2}$. Many candidates had difficulty in simplification of expressions involving root sign. Another common mistake was $\int_0^{\frac{\pi}{4}}\frac{\tan\phi}{\sqrt{1+2\cos^2\phi}}d\phi=\int_0^1\frac{t}{1+\frac{2}{1+t^2}}\cdot\frac{1}{1+t^2}dt$.

12. (a) (i) Good. A minority of candidates wrongly expressed the time taken from P to Q as $7x$ or $\frac{7}{x}$, instead of the correct expression $\frac{x}{7}$.

(ii) Poor. About half of the candidates knew the process in finding the minimum. After the correct proof in the first part, some candidates gave the decimal values of x but these values could not guarantee the exact value of QB as given in the question. Also, a few candidates wrongly rejected $x=40-\frac{5\sqrt{6}}{2}$.

(b) (i) Very Poor. Only about a third of the candidates obtained marks in this part. Among them, most candidates were successful in finding $\sin\beta$ and/or $\cos\beta$. Then, many candidates did not employ sine formula to the appropriate triangle. Also, a few candidates wrongly took ΔAMB as a right-angled triangle.

(ii) Very Poor. Only about a quarter of the candidates obtained marks in this part. Among them, most candidates knew the process in differentiating the expression in (b)(i) with respect to time. One common error was the failure to recognize that $\frac{dMB}{dt}$ was negative, and hence they wrongly gave a positive value as the final answer.

Question	Part		Comments
13.	(a)	(i)	Satisfactory. Some candidates mixed up trace with transpose. Some candidates evaluated MN and NM correctly but did not find $\mathrm{tr}(MN)$ and $\mathrm{tr}(NM)$. Other mistakes included $\mathrm{tr}(MN)=(a+d)\,\mathrm{tr}\begin{pmatrix} e & f \\ g & h \end{pmatrix}$ and $\mathrm{tr}(MN)=\mathrm{tr}(M)\mathrm{tr}(N)$.
		(ii)	Very Poor. Many candidates committed mistakes like $\mathrm{tr}(BAB^{-1})=\mathrm{tr}(BB^{-1}A)$. Some candidates wrongly used (a)(i) to claim that $A=AB^{-1}B=BAB^{-1}=\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$. No marks for accuracy were given though some candidates could use this incorrect finding to derive the answer.
		(iii)	Very Poor. Many candidates wrongly claiming that $A=\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$ at (a)(ii) gave the answer without steps. Some candidates wrote $\begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix}=4$. Some claimed that $\lvert A\rvert=\mathrm{tr}(A)$. Some candidates just recited solution of past papers and wrote things like $(BAB^{-1})^n=BA^nB^{-1}$ and used this to conclude that $\lvert A\rvert=3$. Quite a number of candidates did (a)(ii) and (iii) by expanding $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ in a lengthy manner and some obtained correct answers finally.
	(b)	(i)	Very Poor. Many candidates wrongly concluded from $\begin{pmatrix} p-\lambda_1 & q \\ r & s-\lambda_1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=0$ that $\begin{pmatrix} p-\lambda_1 & q \\ r & s-\lambda_1 \end{pmatrix}=0$, hence $p=s=\lambda_1$ and $q=r=0$. Some candidates wrote wrong things like $\begin{vmatrix} p-\lambda_1 & q \\ r & s-\lambda_1 \end{vmatrix}\begin{vmatrix} x \\ y \end{vmatrix}=0$ and $\begin{pmatrix} p & q \\ r & s \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}^{-1}=\lambda\begin{pmatrix} x \\ y \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}^{-1}$.
		(ii)	Very Poor. Many candidates failed to prove the result because of mistakes in previous parts, such as using $\lambda_1=\lambda_2$ or $p=s=\lambda_1$, $q=r=0$. Some candidates tried to prove the result by considering the sum and the product of roots, but most of them could not complete the argument.
	(c)		Very Poor. Over 90% of the candidates got no marks in this part. Some candidates carrying mistakes from previous parts claimed that $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\lambda\begin{pmatrix} x \\ y \end{pmatrix}$ and concluded that $\lambda=1$ or 3 .
14.	(a)	(i)	Satisfactory. Some candidates lost mark as they did not mention $\mathbf{a}\cdot\mathbf{b}=0$ when deriving $(\mathbf{p}-\mathbf{a})\cdot(\mathbf{p}-\mathbf{b})=\mathbf{p}\cdot\mathbf{p}-(\mathbf{a}+\mathbf{b})\cdot\mathbf{p}$.
		(ii)	Poor. Many candidates were not able to link the result in (a)(i) with $\overrightarrow{AP}\cdot\overrightarrow{BP}+\overrightarrow{BP}\cdot\overrightarrow{CP}+\overrightarrow{CP}\cdot\overrightarrow{AP}=0$. Some candidates duplicated their effort in (a)(i) by considering $\overrightarrow{AP}\cdot\overrightarrow{BP}+\overrightarrow{BP}\cdot\overrightarrow{CP}+\overrightarrow{CP}\cdot\overrightarrow{AP}=(\mathbf{p}-\mathbf{a})\cdot(\mathbf{p}-\mathbf{b})+(\mathbf{p}-\mathbf{b})\cdot(\mathbf{p}-\mathbf{c})+(\mathbf{p}-\mathbf{c})\cdot(\mathbf{p}-\mathbf{a})$.
		(iii)	Very Poor. Quite a number of candidates made the following wrong argument: $\mathbf{p}\cdot\mathbf{p}=2\mathbf{p}\cdot\mathbf{d}$ $\mathbf{p}\cdot\mathbf{p}-\mathbf{p}\cdot\mathbf{d}=\mathbf{p}\cdot\mathbf{d}$ $\mathbf{p}\cdot(\mathbf{p}-\mathbf{d})=\mathbf{p}\cdot\mathbf{d}$ $\mathbf{p}-\mathbf{d}=\mathbf{d}$ $\therefore \lvert\mathbf{p}-\mathbf{d}\rvert=\lvert\mathbf{d}\rvert$ In explaining the last part, most candidates did not know the meaning of fixed radius and did not realise that $\lvert\overrightarrow{DP}\rvert=$ the distance between D and P .

Question	Part	Comments
(b)	(i)	Very Poor. As most candidates did not have a clear understanding of the geometrical meaning in the previous parts, they were not able to use $PD=OD$ to draw the conclusion.
	(ii)	Very Poor. Very few candidates attempted this part. Among those who did, many were not able to make use of the geometrical meaning of cross product or the concept of normal vector to the plane.

General comments and recommendations

1. Candidates should read all the instructions on the cover page of Question-Answer Book carefully. They should show the essential steps in arriving at the answers, such as clear indication of $\lim_{h\to 0}\frac{\sin h}{h}=1$ in Question 1, the factorization of quadratic polynomial in Question 3, rationalization of algebraic expression in Question 11 and simplification of complicated expression, otherwise marks may be deducted.

2. Candidates should plan their time in order to answer all the questions within her/his competence.

3. Candidates should read the questions carefully and understand the questions before attempting them. For instance, in Question 5, since the given sign table provided sufficient information to determine the x-values of the required points in part (a), there was no need to differentiate the function though that function was explicitly stated. And for many questions requiring using the result of the previous part, candidates should find the linkage between the corresponding expressions and followed by appropriate process. Otherwise, marks would be deducted for the questions where alternative method was not allowed.

4. Candidates are expected to be familiar with the finding of asymptotes and the required presentation.

5. In calculus, candidates should have a good grasp of basic concepts, formulas and workings. They should
   - understand that the negative rate of change shows the decrease of the quantity with respect to time;
   - add the arbitrary constant to the answers in indefinite integral;
   - not miss the absolute value sign in $\int\frac{1}{x}\,dx=\ln\lvert x\rvert+C$;
   - not miss π in the formula of volume of revolution;
   - change the limits in the definite integration by substitution.

6. In vector, candidates should
   - write in appropriate notation such as the vector sign, scalar and vector multiplication signs;
   - note that $\mathbf{a}\cdot\mathbf{b}=\mathbf{a}\cdot\mathbf{c}$ did not imply $\mathbf{b}=\mathbf{c}$;
   - need to show perpendicularity of two vectors in order to have their dot product equal to zero;
   - understand the geometrical interpretation of vector expressions, especially those concerning cross products.

7. In matrix, candidates should be aware that matrix multiplication is not commutative. They should be more careful in the basic operation of matrices since the minor errors might not be reflected in the subsequent process.

8. In system of equations, candidates are expected to be familiar with the different conditions of solution and their related properties with the corresponding coefficient matrix or augmented matrix.

9. Candidates should know that they are expected to find numerical values, even in the intermediate steps, in the form of exact values unless otherwise stated. Marks may be deducted if the final answers were obtained by employing guessing or rounding of numerical values from the calculator.