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香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2021年香港中學文憑考試
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2021

數學 延伸部分 單元二（代數與微積分）
MATHEMATICS EXTENDED PART MODULE 2 (ALGEBRA AND CALCULUS)

評卷參考
MARKING SCHEME

Hong Kong Diploma of Secondary Education Examination
Mathematics Extended Part Module 2 (Algebra and Calculus)

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

5. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

6. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

Solution	Marks	Remarks
1. $f'(x)$		
$= \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$		
$= \lim_{h\to 0}\dfrac{\dfrac{1}{3(x+h)^2+4}-\dfrac{1}{3x^2+4}}{h}$	1M	
$= \lim_{h\to 0}\dfrac{(3x^2+4)-(3(x+h)^2+4)}{h(3(x+h)^2+4)(3x^2+4)}$		
$= \lim_{h\to 0}\dfrac{-3h(2x+h)}{h(3(x+h)^2+4)(3x^2+4)}$	1M	
$= \lim_{h\to 0}\dfrac{-3(2x+h)}{(3(x+h)^2+4)(3x^2+4)}$	1M	withhold 1M if this step is skipped
$= \dfrac{-6x}{(3x^2+4)^2}$	1A	
	----------(4)	
2. Note that $\sum_{k=1}^{1}(3k^5+k^3)=4=\dfrac{1^3(1+1)^3}{2}$.		
Therefore, the statement is true for $n=1$.	1	
Assume that $\sum_{k=1}^{m}(3k^5+k^3)=\dfrac{m^3(m+1)^3}{2}$, where m is a positive integer.	1M	
$\sum_{k=1}^{m+1}(3k^5+k^3)$		
$=\sum_{k=1}^{m}(3k^5+k^3)+3(m+1)^5+(m+1)^3$		
$=\dfrac{m^3(m+1)^3}{2}+3(m+1)^5+(m+1)^3$ (by induction assumption)	1M	for using induction assumption
$=\dfrac{m^3(m+1)^3+6(m+1)^5+2(m+1)^3}{2}$		
$=\dfrac{(m+1)^3(m^3+6(m+1)^2+2)}{2}$	1M	
$=\dfrac{(m+1)^3(m^3+6m^2+12m+8)}{2}$		
$=\dfrac{(m+1)^3(m+2)^3}{2}$		
So, the statement is true for $n=m+1$ if it is true for $n=m$.		
By mathematical induction, the statement is true for all positive integers n .	1	
	----------(5)	

Solution	Marks	Remarks
3. (a) $(1-4x)^n$		
$= 1 - n(4x) + \frac{n(n-1)}{2}(4x)^2 - \cdots + (-1)^n(4x)^n$	1M	
$\frac{n(n-1)}{2}(4^2) = 240$	1M	
$n^2 - n - 30 = 0$		
$n = 6$ or $n = -5$ (rejected)		
Thus, we have $n = 6$.	1A	
(b) $\left(1+\frac{2}{x}\right)^5$		
$= 1 + 5\left(\frac{2}{x}\right) + 10\left(\frac{2}{x}\right)^2 + 10\left(\frac{2}{x}\right)^3 + 5\left(\frac{2}{x}\right)^4 + \left(\frac{2}{x}\right)^5$	1M	either one
$= 1 + \frac{10}{x} + \frac{40}{x^2} + \frac{80}{x^3} + \frac{80}{x^4} + \frac{32}{x^5}$		
$(1-4x)^6$		
$= 1 - 6(4x) + 15(4x)^2 - 20(4x)^3 + 15(4x)^4 - 6(4x)^5 + (4x)^6$		
$= 1 - 24x + 240x^2 - 1\,280x^3 + 3\,840x^4 - 6\,144x^5 + 4\,096x^6$		
The coefficient of x^4		
$= (1)(3\,840) + (10)(-6\,144) + (40)(4\,096)$	1M	withhold 1M if this step is skipped
$= 106\,240$	1A	
	----------(6)	
4. (a) $\cos 2x + \cos 4x + \cos 6x$		
$= 2\cos^2 x - 1 + \cos 4x + \cos 6x$	1M	
$= 2\cos^2 x - 1 + 2\cos 5x \cos x$	1M	either one
$= 2\cos x(\cos x + \cos 5x) - 1$		
$= 2\cos x(2\cos 3x \cos 2x) - 1$		
$= 4\cos x \cos 2x \cos 3x - 1$	1	
(b) $\cos 4\theta + \cos 8\theta + \cos 12\theta = -1$		
$4\cos 2\theta \cos 4\theta \cos 6\theta - 1 = -1$ (by putting $x = 2\theta$ in (a))	1M	
$\cos 2\theta \cos 4\theta \cos 6\theta = 0$		
$\cos 2\theta = 0$, $\cos 4\theta = 0$ or $\cos 6\theta = 0$	1M	
$\theta = \frac{\pi}{12}$, $\theta = \frac{\pi}{8}$, $\theta = \frac{\pi}{4}$, $\theta = \frac{3\pi}{8}$ or $\theta = \frac{5\pi}{12}$	1A	for all correct
$\cos 4\theta + \cos 8\theta + \cos 12\theta = -1$		
$1 + \cos 8\theta + \cos 4\theta + \cos 12\theta = 0$		
$2\cos^2 4\theta + 2\cos 8\theta \cos 4\theta = 0$		
$\cos 4\theta(\cos 4\theta + \cos 8\theta) = 0$	1M	
$2\cos 2\theta \cos 4\theta \cos 6\theta = 0$		
$\cos 2\theta = 0$, $\cos 4\theta = 0$ or $\cos 6\theta = 0$	1M	
$\theta = \frac{\pi}{12}$, $\theta = \frac{\pi}{8}$, $\theta = \frac{\pi}{4}$, $\theta = \frac{3\pi}{8}$ or $\theta = \frac{5\pi}{12}$	1A	for all correct
	----------(6)	

Solution	Marks	Remarks
5. (a) The equation of the vertical asymptote is $x-1=0$.	1A	
Note that $\mathrm{r}(x)=x+1-\dfrac{x-2}{(x-1)^2}$.	1M	
Thus, the equation of the oblique asymptote is $y=x+1$.	1A	f.t.
(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\mathrm{r}(x)$ $=\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x+1-\dfrac{x-2}{(x-1)^2}\right)$		
$=1-\dfrac{(x-1)^2-2(x-2)(x-1)}{(x-1)^4}$	1M	
$=1+\dfrac{x-3}{(x-1)^3}$	1A	
$\dfrac{\mathrm{d}}{\mathrm{d}x}\mathrm{r}(x)$ $=\dfrac{(x-1)^2(3x^2-2x-2)-2(x-1)(x^3-x^2-2x+3)}{(x-1)^4}$	1M	
$=\dfrac{x^3-3x^2+4x-4}{(x-1)^3}$	1A	
(c) Note that $\dfrac{\mathrm{d}^2}{\mathrm{d}x^2}\mathrm{r}(x)=\dfrac{(x-1)^3-3(x-1)^2(x-3)}{(x-1)^6}=\dfrac{-2(x-4)}{(x-1)^4}$. So, we have $\dfrac{\mathrm{d}^2}{\mathrm{d}x^2}\mathrm{r}(x)=0 \iff x=4$.		

x	$(1, 4)$	4	$(4, \infty)$
$\dfrac{\mathrm{d}^2}{\mathrm{d}x^2}\mathrm{r}(x)$	$+$	0	$-$

Solution	Marks	Remarks
(table above)	1M	for testing
Therefore, there is only one point of inflexion of the graph of $y=\mathrm{r}(x)$. Thus, the claim is agreed.	1A -----------(7)	f.t.

Solution	Marks	Remarks
6. (a) $y = e^{2x-6}$		
$\frac{dy}{dx} = 2e^{2x-6}$	1M	
$\left.\frac{dy}{dx}\right\|_{x=3} = 2e^{2(3)-6} = 2$		
The equation of L is		
$y - 1 = \frac{-1}{2}(x-3)$		
$x + 2y - 5 = 0$		
Putting $x = c$ and $y = 0$ in $x + 2y - 5 = 0$, we have $c + 2(0) - 5 = 0$.	1M	
Thus, we have $c = 5$.	1A	
(b) The required area		
$= \int_3^5 \left(e^{2x-6} - \left(\frac{-x}{2} + \frac{5}{2} \right) \right) dx$	1M+1A	
$= \int_3^5 \left(e^{2x-6} + \frac{x}{2} - \frac{5}{2} \right) dx$		
$= \left[\frac{e^{2x-6}}{2} + \frac{x^2}{4} - \frac{5x}{2} \right]_3^5$	1M	
$= \frac{e^4 - 3}{2}$	1A	
	----------(7)	
7. (a) $\int (\ln x)^2 \, dx$		
$= x(\ln x)^2 - \int x \left(\frac{2\ln x}{x} \right) dx$	1M	
$= x(\ln x)^2 - 2\left(x\ln x - \int x \left(\frac{1}{x} \right) dx \right)$	1M	
$= x(\ln x)^2 - 2x\ln x + 2x + \text{constant}$	1A	
(b) The required volume		
$= \int_0^1 \pi \left(\sqrt{x} \ln(x^2+1) \right)^2 dx$	1M	
$= \pi \int_0^1 x \left(\ln(x^2+1) \right)^2 dx$		
$= \frac{\pi}{2} \int_1^2 (\ln u)^2 \, du$ (by letting $u = x^2 + 1$)	1M	
$= \frac{\pi}{2} \left[u(\ln u)^2 - 2u\ln u + 2u \right]_1^2$ (by (a))	1M	for using the result of (a)
$= \pi((\ln 2)^2 - 2\ln 2 + 1)$	1A	
	----------(7)	

Solution	Marks	Remarks
8. (a) Note that $\begin{vmatrix} 1 & d-1 & d+3 \\ 2 & d+2 & -1 \\ 3 & d+4 & 5 \end{vmatrix}$		
$= 5(d+2)+2(d+4)(d+3)+3(-1)(d-1)-3(d+2)(d+3)-10(d-1)-(-1)(d+4)$ $= -d^2-8d+33$	1M	
As (E) has infinitely many solutions, we have $\begin{vmatrix} 1 & d-1 & d+3 \\ 2 & d+2 & -1 \\ 3 & d+4 & 5 \end{vmatrix} = 0$.	1M	
So, we have $-d^2-8d+33=0$. Solving, we have $d=-11$ or $d=3$.		
When $d=-11$, the augmented matrix of (E) is $\left(\begin{array}{ccc\|c} 1 & -12 & -8 & 15 \\ 2 & -9 & -1 & -27 \\ 3 & -7 & 5 & 2 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & -12 & -8 & 15 \\ 0 & -15 & -15 & 57 \\ 0 & -29 & -29 & 43 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & -12 & -8 & 15 \\ 0 & -15 & -15 & 57 \\ 0 & 0 & 0 & 1 \end{array}\right)$	1M	either one (with the alternative below)
Since (E) is consistent, we have $d \neq -11$.	1M	
Therefore, we have $d=3$.	1A	f.t.
Hence, the augmented matrix of (E) is $\left(\begin{array}{ccc\|c} 1 & 2 & 6 & 1 \\ 2 & 5 & -1 & 1 \\ 3 & 7 & 5 & 2 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 2 & 6 & 1 \\ 0 & -1 & 13 & 1 \\ 0 & -1 & 13 & 1 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 2 & 6 & 1 \\ 0 & -1 & 13 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$		
Thus, the solution set of (E) is $\{(3-32t, 13t-1, t) : t \in \mathbf{R}\}$.	1A	
(b) Putting $x=3-32t$, $y=13t-1$ and $z=t$ in $xy+2xz=3$, we have $(3-32t)(13t-1)+2(3-32t)t=3$. Therefore, we have $-480t^2+77t-6=0$.		
Note that $77^2-4(-480)(-6)=-5\,591<0$.	1M	
So, (E) does not have a real solution (x, y, z) satisfying $xy+2xz=3$. Thus, the claim is not correct.	1A	f.t.
	----------(8)	

Solution	Marks	Remarks
9. (a) (i) $\frac{d}{d\theta}\ln(\sec\theta+\tan\theta)$		
$=\left(\frac{1}{\sec\theta+\tan\theta}\right)(\sec\theta\tan\theta+\sec^2\theta)$		
$=\sec\theta$	1A	
(ii) By (a)(i), we have $\frac{d}{d\theta}\ln(\sec\theta+\tan\theta)=\sec\theta$.		
So, we have $\int\sec\theta\,d\theta=\ln(\sec\theta+\tan\theta)+\text{constant}$.	1A	
$\int\sec^3\theta\,d\theta=\sec\theta\tan\theta-\int\tan^2\theta\sec\theta\,d\theta$	1M	
$\int\sec^3\theta\,d\theta=\sec\theta\tan\theta-\int(\sec^2\theta-1)\sec\theta\,d\theta$		
$2\int\sec^3\theta\,d\theta=\sec\theta\tan\theta+\int\sec\theta\,d\theta$		
$\int\sec^3\theta\,d\theta=\frac{1}{2}(\sec\theta\tan\theta+\ln(\sec\theta+\tan\theta))+\text{constant}$	1A	
	----------(4)	
(b) Note that $\int_{-a}^{a}g(x)h(x)dx=-\int_{a}^{-a}g(-x)h(-x)dx=\int_{-a}^{a}g(-x)h(x)dx$.	1M	
$\int_{-a}^{a}g(x)h(x)dx$		
$=\frac{1}{2}\left(\int_{-a}^{a}g(x)h(x)dx+\int_{-a}^{a}g(x)h(x)dx\right)$		
$=\frac{1}{2}\left(\int_{-a}^{a}g(x)h(x)dx+\int_{-a}^{a}g(-x)h(x)dx\right)$		
$=\frac{1}{2}\int_{-a}^{a}(g(x)+g(-x))h(x)dx$		
$=\frac{1}{2}\int_{-a}^{a}h(x)dx$		
$=\frac{1}{2}\left(\int_{-a}^{0}h(x)dx+\int_{0}^{a}h(x)dx\right)$	1M	
$=\frac{1}{2}\left(-\int_{a}^{0}h(-y)dy+\int_{0}^{a}h(x)dx\right)$ (by letting $x=-y$)		
$=\frac{1}{2}\left(\int_{0}^{a}h(-x)dx+\int_{0}^{a}h(x)dx\right)$		
$=\frac{1}{2}\left(\int_{0}^{a}h(x)dx+\int_{0}^{a}h(x)dx\right)$		
$=\frac{1}{2}\left(2\int_{0}^{a}h(x)dx\right)$		
$=\int_{0}^{a}h(x)dx$	1	
	----------(3)	

Solution	Marks	Remarks
(c) Let $g(x)=\dfrac{3^x}{3^x+3^{-x}}$ and $h(x)=\dfrac{x^2}{\sqrt{x^2+1}}$ for all $x\in\mathbf{R}$.		
Note that $g(x)+g(-x)=\dfrac{3^x}{3^x+3^{-x}}+\dfrac{3^{-x}}{3^{-x}+3^{-(-x)}}=1$ and $h(-x)=h(x)$.	1M	withhold 1M if this step is skipped
$\displaystyle\int_{-1}^{1}\frac{3^x x^2}{(3^x+3^{-x})\sqrt{x^2+1}}\mathrm{d}x$		
$\displaystyle=\int_{-1}^{1}g(x)h(x)\,\mathrm{d}x$		
$\displaystyle=\int_{0}^{1}h(x)\,\mathrm{d}x$ (by putting $a=1$ in (b))	1M	
$\displaystyle=\int_{0}^{1}\frac{x^2}{\sqrt{x^2+1}}\mathrm{d}x$		
$\displaystyle=\int_{0}^{\frac{\pi}{4}}\frac{\tan^2\theta\sec^2\theta}{\sqrt{\tan^2\theta+1}}\mathrm{d}\theta$ (by letting $x=\tan\theta$)	1M	
$\displaystyle=\int_{0}^{\frac{\pi}{4}}\tan^2\theta\sec\theta\,\mathrm{d}\theta$		
$\displaystyle=\int_{0}^{\frac{\pi}{4}}(\sec^3\theta-\sec\theta)\,\mathrm{d}\theta$		
$\displaystyle=\int_{0}^{\frac{\pi}{4}}\sec^3\theta\,\mathrm{d}\theta-\int_{0}^{\frac{\pi}{4}}\sec\theta\,\mathrm{d}\theta$		
$\displaystyle=\left[\frac{1}{2}(\sec\theta\tan\theta+\ln(\sec\theta+\tan\theta))\right]_0^{\frac{\pi}{4}}-\left[\ln(\sec\theta+\tan\theta)\right]_0^{\frac{\pi}{4}}$ (by (a)(ii))	1M	for using the results of (a)(ii)
$\displaystyle=\frac{1}{2}\left(\sqrt{2}-\ln(\sqrt{2}+1)\right)$	1A	
	-----------(5)	

Solution	Marks	Remarks
10. (a) PQ		
$=\sqrt{u^2+36}+\sqrt{(20-u)^2+16}$	1M	
$\frac{\mathrm{d}PQ}{\mathrm{d}u}$		
$=\frac{u}{\sqrt{u^2+36}}-\frac{20-u}{\sqrt{(20-u)^2+16}}$	1M	
$=\frac{u\sqrt{(20-u)^2+16}-(20-u)\sqrt{u^2+36}}{\sqrt{u^2+36}\sqrt{(20-u)^2+16}}$		
$=\frac{-20(u-12)(u-60)}{\sqrt{u^2+36}\sqrt{(20-u)^2+16}\left(u\sqrt{(20-u)^2+16}+(20-u)\sqrt{u^2+36}\right)}$		
For $\frac{\mathrm{d}PQ}{\mathrm{d}u}=0$, we have $u=12$.	1M	
Thus, we have $a=12$.	1A	
	----------(4)	
(b) (i) Let A square units be the area of the rectangle $PQSR$.		
Then, we have $A=u\left(\sqrt{u^2+36}+\sqrt{(20-u)^2+16}\right)$.	1M	
$\frac{\mathrm{d}A}{\mathrm{d}u}$		
$=u\left(\frac{u}{\sqrt{u^2+36}}-\frac{20-u}{\sqrt{(20-u)^2+16}}\right)+\sqrt{u^2+36}+\sqrt{(20-u)^2+16}$	1M	
Therefore, we have $\left.\frac{\mathrm{d}A}{\mathrm{d}u}\right\rvert_{u=12}=10\sqrt{5}\neq 0$.	1M	
Hence, A does not attain its minimum value when $u=12$. Thus, the claim is disagreed.	1A	f.t.
(ii) Since $OP=\sqrt{u^2+u^2+36}$, we have $OP=\sqrt{2u^2+36}$.		
At time t minutes, we have $\frac{\mathrm{d}OP}{\mathrm{d}t}=\frac{2u}{\sqrt{2u^2+36}}\left(\frac{\mathrm{d}u}{\mathrm{d}t}\right)$.	1M	
As $28=\frac{2(12)}{\sqrt{2(12^2)+36}}\left(\left.\frac{\mathrm{d}u}{\mathrm{d}t}\right\rvert_{u=12}\right)$, we have $\left.\frac{\mathrm{d}u}{\mathrm{d}t}\right\rvert_{u=12}=21$.	1M	
Let w units be the perimeter of the rectangle $PQSR$.		
Then, we have $w=2\left(u+\sqrt{u^2+36}+\sqrt{(20-u)^2+16}\right)$.	1M	
Therefore, we have $\frac{\mathrm{d}w}{\mathrm{d}t}=2\left(1+\frac{u}{\sqrt{u^2+36}}-\frac{(20-u)}{\sqrt{(20-u)^2+16}}\right)\frac{\mathrm{d}u}{\mathrm{d}t}$.	1M	
$\left.\frac{\mathrm{d}w}{\mathrm{d}t}\right\rvert_{u=12}$		
$=2\left(1+\frac{12}{\sqrt{180}}-\frac{8}{\sqrt{80}}\right)(21)$		
$=42$	1A	
Thus, the required rate of change is 42 units per minute.		
	----------(9)	

Solution	Marks	Remarks
11. (a) Note that $P^{-1}=\begin{pmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{pmatrix}$.	1A	
PAP^{-1} $=\begin{pmatrix}\sin\theta & \cos\theta\\ -\cos\theta & \sin\theta\end{pmatrix}\begin{pmatrix}\alpha & \beta\\ \beta & -\alpha\end{pmatrix}\begin{pmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{pmatrix}$		
$=\begin{pmatrix}\alpha\sin\theta+\beta\cos\theta & \beta\sin\theta-\alpha\cos\theta\\ -\alpha\cos\theta+\beta\sin\theta & -\beta\cos\theta-\alpha\sin\theta\end{pmatrix}\begin{pmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{pmatrix}$	1M	
$=\begin{pmatrix}\alpha\sin^2\theta+2\beta\sin\theta\cos\theta-\alpha\cos^2\theta & -2\alpha\sin\theta\cos\theta-\beta\cos^2\theta+\beta\sin^2\theta\\ -2\alpha\sin\theta\cos\theta-\beta\cos^2\theta+\beta\sin^2\theta & -\alpha\sin^2\theta-2\beta\sin\theta\cos\theta+\alpha\cos^2\theta\end{pmatrix}$		
$=\begin{pmatrix}-\alpha\cos2\theta+\beta\sin2\theta & -\beta\cos2\theta-\alpha\sin2\theta\\ -\beta\cos2\theta-\alpha\sin2\theta & \alpha\cos2\theta-\beta\sin2\theta\end{pmatrix}$	1	
	----------(3)	
(b) (i) By (a), we have $PBP^{-1}=\begin{pmatrix}-\cos2\theta+\sqrt{3}\sin2\theta & -\sqrt{3}\cos2\theta-\sin2\theta\\ -\sqrt{3}\cos2\theta-\sin2\theta & \cos2\theta-\sqrt{3}\sin2\theta\end{pmatrix}$.		
For $PBP^{-1}=\begin{pmatrix}\lambda & 0\\ 0 & \mu\end{pmatrix}$, we have $-\sqrt{3}\cos2\theta-\sin2\theta=0$.	1M	
So, we have $\tan2\theta=-\sqrt{3}$.		
Since $\frac{\pi}{2}<\theta<\pi$, we have $\theta=\frac{5\pi}{6}$.	1A	
(ii) Since $\theta=\frac{5\pi}{6}$, we have $\lambda=-2$ and $\mu=2$.		
So, we have $PBP^{-1}=\begin{pmatrix}-2 & 0\\ 0 & 2\end{pmatrix}$.		
Therefore, we have $B=P^{-1}\begin{pmatrix}-2 & 0\\ 0 & 2\end{pmatrix}P$.		
B^n $=\left(P^{-1}\begin{pmatrix}-2 & 0\\ 0 & 2\end{pmatrix}P\right)^n$		
$=P^{-1}\begin{pmatrix}-2 & 0\\ 0 & 2\end{pmatrix}^nP$	1M	
$=P^{-1}\begin{pmatrix}(-2)^n & 0\\ 0 & 2^n\end{pmatrix}P$	1M	
$=\begin{pmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ \frac{-\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}(-2)^n & 0\\ 0 & 2^n\end{pmatrix}\begin{pmatrix}\frac{1}{2} & \frac{-\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}$	1M	
$=\frac{1}{4}\begin{pmatrix}(-1)^n2^n & \sqrt{3}(2^n)\\ \sqrt{3}(-1)^{n+1}2^n & 2^n\end{pmatrix}\begin{pmatrix}1 & -\sqrt{3}\\ \sqrt{3} & 1\end{pmatrix}$		
$=2^{n-2}\begin{pmatrix}(-1)^n+3 & \sqrt{3}(-1)^{n+1}+\sqrt{3}\\ \sqrt{3}(-1)^{n+1}+\sqrt{3} & 3(-1)^n+1\end{pmatrix}$	1	

Solution	Marks	Remarks
(iii) B^{555}		
$=2^{555-2}\begin{pmatrix}(-1)^{555}+3 & \sqrt{3}(-1)^{556}+\sqrt{3}\\ \sqrt{3}(-1)^{556}+\sqrt{3} & 3(-1)^{555}+1\end{pmatrix}$ (by (b)(ii))	1M	for using (b)(ii)
$=2^{553}\begin{pmatrix}2 & 2\sqrt{3}\\ 2\sqrt{3} & -2\end{pmatrix}$		
$(B^{-1})^{555}$		
$=(B^{555})^{-1}$	1M	
$=\dfrac{1}{2^{553}}\begin{pmatrix}2 & 2\sqrt{3}\\ 2\sqrt{3} & -2\end{pmatrix}^{-1}$		
$=\dfrac{1}{2^{553}}\begin{pmatrix}\frac{1}{8} & \frac{\sqrt{3}}{8}\\ \frac{\sqrt{3}}{8} & \frac{-1}{8}\end{pmatrix}$		
$=\dfrac{1}{2^{556}}\begin{pmatrix}1 & \sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}$	1A	
$B=\begin{pmatrix}1 & \sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}$		
$B^2=\begin{pmatrix}1 & \sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}\begin{pmatrix}1 & \sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}=4\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=2^2 I$		
$B^4=(B^2)^2=(2^2 I)^2=2^4 I$	1M	
$B^6=2^6 I$		
$B^{554}=2^{554} I$		
B^{-1}		
$=\dfrac{1}{-4}\begin{pmatrix}-1 & -\sqrt{3}\\ -\sqrt{3} & 1\end{pmatrix}$		
$=\begin{pmatrix}\frac{1}{4} & \frac{\sqrt{3}}{4}\\ \frac{\sqrt{3}}{4} & \frac{-1}{4}\end{pmatrix}$		
$(B^{-1})^{555}$		
$=(B^{555})^{-1}$	1M	
$=(B(B^{554}))^{-1}$		
$=(2^{554}B)^{-1}$		
$=\dfrac{1}{2^{554}}B^{-1}$		
$=\dfrac{1}{2^{556}}\begin{pmatrix}1 & \sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}$	1A	
	----------(9)	

Solution	Marks	Remarks
12. (a) (i) Note that $\overrightarrow{AB}=(12-t)\mathbf{i}-(s+14)\mathbf{j}-(2+s)\mathbf{k}$.		
Since $\overrightarrow{AB}$ is parallel to $5\mathbf{i}-4\mathbf{j}-2\mathbf{k}$, we have		
$\frac{12-t}{5}=\frac{-(s+14)}{-4}=\frac{-(2+s)}{-2}$	1M	
Solving, we have $s=10$ and $t=-18$.	1A	for both correct
(ii) Note that $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 30 & -24 & -12\\ 30 & -30 & 0\end{vmatrix}=-360\mathbf{i}-360\mathbf{j}-180\mathbf{k}$.	1M	
The area of ΔABC		
$=\frac{1}{2}\left\lvert\overrightarrow{AB}\times\overrightarrow{AC}\right\rvert$	1M	
$=\frac{1}{2}\sqrt{360^2+360^2+180^2}$		
$=270$	1A	
(iii) Note that $\overrightarrow{AD}=36\mathbf{i}-2\mathbf{j}+4\mathbf{k}$.		
The required volume		
$=\frac{1}{6}\left\lvert(\overrightarrow{AB}\times\overrightarrow{AC})\cdot\overrightarrow{AD}\right\rvert$	1M	
$=\frac{1}{6}\lvert(-360\mathbf{i}-360\mathbf{j}-180\mathbf{k})\cdot(36\mathbf{i}-2\mathbf{j}+4\mathbf{k})\rvert$		
$=\frac{1}{6}\lvert(-360)(36)+(-360)(-2)+(-180)(4)\rvert$		
$=\frac{1}{6}\lvert-12\,960\rvert$		
$=2\,160$	1A	
(iv) Let d be the shortest distance from D to Π .		
$\frac{1}{3}(270)d=2\,160$	1M	
$d=24$	1A	
Thus, the shortest distance from D to Π is 24 .		
	----------(9)	
(b) Note that $\overrightarrow{AB}\times\overrightarrow{AC}=-360\mathbf{i}-360\mathbf{j}-180\mathbf{k}$ and $(\overrightarrow{AB}\times\overrightarrow{AC})\cdot\overrightarrow{AD}<0$.		
$\overrightarrow{DE}$		
$=\frac{24}{\sqrt{(-360)^2+(-360)^2+(-180)^2}}(-360\mathbf{i}-360\mathbf{j}-180\mathbf{k})$	1M	
$=-16\mathbf{i}-16\mathbf{j}-8\mathbf{k}$		
Also note that $\overrightarrow{EA}=-20\mathbf{i}+18\mathbf{j}+4\mathbf{k}$ and $\overrightarrow{EB}=10\mathbf{i}-6\mathbf{j}-8\mathbf{k}$.	1M	for either one
Let M be the mid-point of AB .		
$\overrightarrow{EM}=\frac{1}{2}(\overrightarrow{EA}+\overrightarrow{EB})=-5\mathbf{i}+6\mathbf{j}-2\mathbf{k}$		
$\overrightarrow{EM}\cdot\overrightarrow{AB}=(-5)(30)+(6)(-24)+(-2)(-12)=-270\neq0$	1M	
So, EM is not perpendicular to AB .		
Thus, E is not the circumcentre of ΔABC .	1A	f.t.
	----------(4)	